*Reliability theorem,*
*DGPS system,*
*System Availability*

Cezary SPECHT[1]

## RELIABILITY THEOREM APPLICATION FOR ACCURACY STATISTICS DETERMINATION OF DIFFERENTIAL GLOBAL POSITIONING SYSTEM

*The accuracy estimation method of determining the position coordinates of temporary navigation systems is well known both in navigation and geodesy as well. The estimation of errors measurements statistics being the random variables enables evaluation some probabilities which are measures of accuracy of the position systems. There are presented the suggestion of three new measures of accuracy of determining the coordinates related to reliability characteristics in that article. In contrast to the common applied approach in which the single measurement error is a random variable, in the presented here approach the distributions of working times and times of failures were taken under statistics estimation. The model of availability was detailed presented. Is enabled to define some new statistics measures related to the navigational solution. Proposed method were applied to DGPS system position statistics, where two campaign (2005, 2009) were compared.*

## WYKORZYSTANIE TEORII NIEZAWODNOŚCI DLA WYZNACZENIA STATYSTYK DOKŁADNOŚCIOWYCH RÓŻNICOWEGO GLOBALNEGO SYSTEMU POZYCYJNEGO

*Metoda określenia dokładności wyznaczenia pozycji współczesnych systemów nawigacyjnych jest dziś powszechnie znana zarówno w nawigacji jak i geodezji. Określenie statystyk błędów pomiarów – będących zmiennymi losowymi - umożliwia wyznaczanie parametrów statystycznych, które określają dokładność system. W artykule zaproponowano trzy nowe charakterystyki dokładnościowe odniesione do charakterystyk niezawodnościowych. W przeciwieństwie do aktualnego podejścia proponowany model odnosi zmienne losowe do czasów pracy i awarii systemu. Model dostępności został zaprezentowany w sposób szczegółowy. Proponowane podejście umożliwia zdefiniowanie nowych statystyk odniesionych do procesu nawigacji. Opracowana metoda została zastosowana do rozwiązań pozycyjnych systemu DGPS, gdzie porównano dwie kampanie pomiarowe z lat: 2005 i 2009.*

[1]Naval Academy in Gdynia, c.specht@geodezja.pl, tel. 58 6262632

## 1. INTRODUCTION

The problem of estimation and analysis of statistics distributions of the errors of determining the coordinates of the navigation and geodesic (measurements) systems is well known and described in the literature [1, 4]. The distributions and the parameters of the random variable – the error of determining the coordinates (density function, distribution function, the expected value, variance, moments etc.) should be known to each graduate of the high school. They enable creating so called the accuracy characteristics of the navigation systems.

Notice, that the problem of determining the coordinates of position for the needs of navigation, considered only in terms of the measurement error seems to be solved. Its realization, less or more precisely, is a function of the applied technical solution. In that case the reliability characteristics of navigational system like availability, reliability, continuity or likelihood get an important meaning [2]. We can state that the likelihood [3] and the characteristics: availability reliability and continuity considered on various levels of the structure of the radio-navigational position systems seem to be the main directions of research in navigation. Generally, the characteristics of the temporary navigational system decide on the quality of it. Those characteristics are parameters, which enable the unambiguous estimation.

So far perceiving of those two groups of criteria of estimation of the navigational systems (accuracy and reliability characteristics) has differentiated them due to the different methods of statistical and probabilistic inference. Very often the reliability problems are restricted to the popular understanding of them i.e. to the technical or exploitation aspects of devices. The three new probabilities introducing some definitions are proposed below:

**availability of determining position** – the probability that in any moment of $t$ the error of determining the coordinates of the position $\delta_n < U$, $n = 1,2,...$ ;

**reliability of determining the position** - the probability that in the given time interval $[t, \tau)$ the error of determining the coordinates of the position $\delta_n < U, n = 1,2,...$, when $U$ is arbitrary fixed;

**continuity of determining the position** – the conditional probability that in the given time interval $[t, \tau)$ the error $\delta_n < U$; $n = 1,2,...$ given that in the moment $t$, $\delta_t < U$ holds.

## 2. THE WORKING STATES OF NAVIGATION SYSTEM

Let the error of determining the coordinates of the position of any navigational system in the function of time be a variable taking its values from the given interval of errors $\langle 0, \infty)$ (Fig. 1).

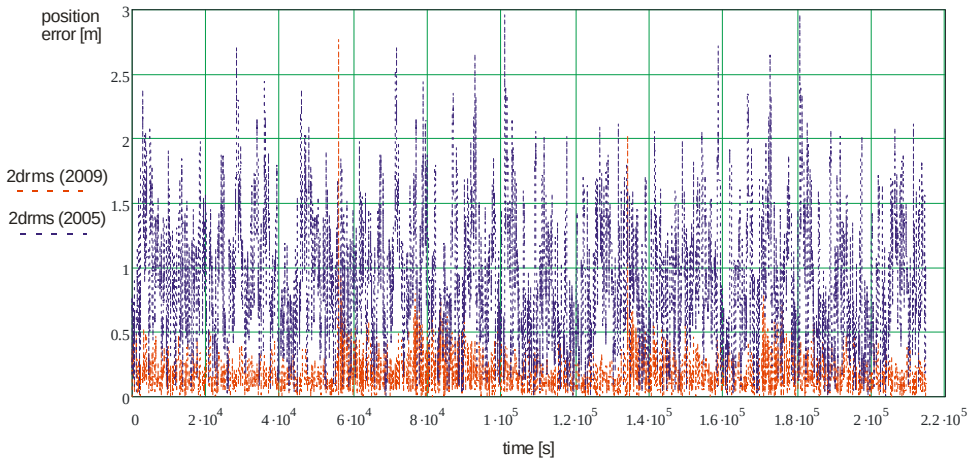


*Fig. 1. The error of determining the coordinates of the position as a function of time (DGPS system in 2005 and 2009)*

Assume that the process of determining the coordinates of the position is alternating with the renew in the sense of the general reliability theory. Then we can recognize two states: the working one – the state where the error $\delta_n \le U$ for $n = 1,2,...$ and the state of failure where $\delta_n > U$. Let $X_1, X_2, ...$ be the working times while $Y_1, Y_2, ...$ are the times of failures. Hence the moments $Z_n^{'} = X_1 + Y_1 + X_2 + Y_2 + ... + Y_{n-1} + X_n$, $n = 1,2,...,$ are the moments of failures and $Z_n^{''} = Z_n^{'} + Y_n$, are the moments of renewal. Assume also that the random variables $X_i, Y_i$, $i = 1,2,...$ are independent and that the working times and times of failures have the same distributions.

The distribution functions of working times and the times of failures are right-hand continuous and then a certain value of a parameter

$$U = \begin{cases} 1\,\text{m} \\ 1.5\,\text{m} \end{cases} \tag{1}$$

the working states (binary: 1) and states of failures (binary: 0) of the process can be presented as follows:

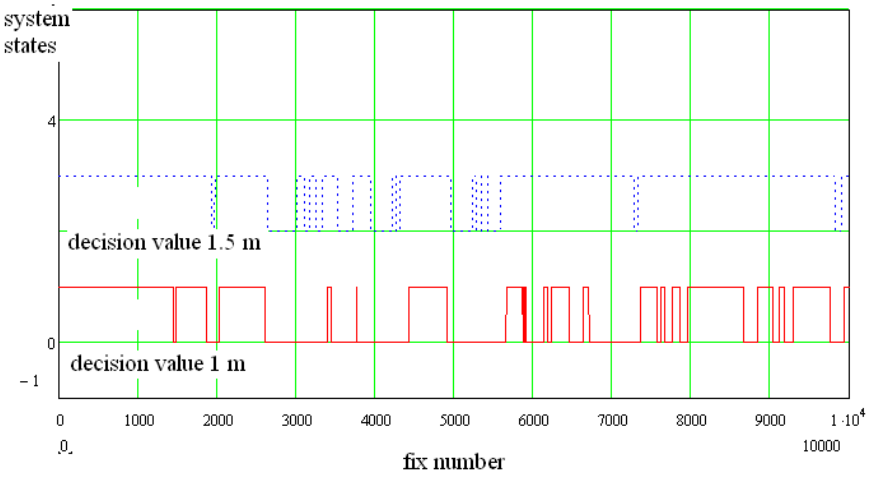


*Fig. 2. Working states [values: 1, 3] and states of failures [values; 0, 2] of the reliability process of the DGPS system position (in 2005) for* $U = 1\,\mathrm{m}$ *and* $U = 1.5\,\mathrm{m}$

From the above analysis it follows that for different values of $U$ the working times $X_n$ and the times of failure $Y_n$ are also different. The discrete moments $Z_n'$ and $Z_n''$ behave similarly. Now we define the analytical form of the distributions of the variables $X_n$ and $Y_n$ as

$$P(Y_i \le y) = G(y) \text{ for } i = 1,2,..., \tag{2}$$

where: $F(x)$ means the distribution function of $X_n$

$G(y)$ means the distribution function of $Y_n$

Introduce also the notations of the expected value and the variance as follows

$$E(X_i) = E(X), E(Y_i) = E(Y) \tag{3}$$

$$V(X_i) = \sigma_1^2 \ , \ V(Y_i) = \sigma_2^2 \ \ i = 1,2,.... \tag{4}$$

where: $E(X_i)$ means the expected value of the working time

$E(Y_i)$ means the expected value of the time of failure

We also admit that $\sigma_1^2 + \sigma_2^2 > 0$.

## 3. EXPONENTIAL MODEL OF POSITION AVAILABILITY

On the basis of the above mentioned assumptions we define the reliability process in which the relation between the single measurement error $\delta_n$ and the parameter $U$ decide

about its state (work or failure). Let $\alpha(t)$ be the binary interpretation of the reliability state of the process :

$$\alpha(t)=\begin{cases} 1, & Z''_n \le t < Z'_{n+1} \\ 0, & Z'_{n+1} \le t < Z''_{n+1} \end{cases} \text{ for } n=0,1,... \tag{5}$$

The state $\alpha(t)=1$ means that in the moment $t$ the error of the single measurement is less or equal than $U$. In the opposite case for $\delta_n > U$, the system is in the state of failure. Due to the definition of the availability, the probability of the fact that in any point of time $t$, the error of determining the coordinates of the position $\delta_n$ be less than the arbitrary taken value of $U$, will be called the availability of a certain value of the error of determining the coordinates and will be denoted as

$$A(t) = P[\delta(t) \le U] \tag{6}$$

The final form of $A(t)$ could be presented as [5]

$$A(t)=1-F(t)+\int_0^t [1-F(t-x)]dH_\Phi(x) \tag{7}$$

Where, $H_\Phi(x)=\sum_{n=1}^{\infty}\Phi_n(x)$ is a renewal function of stream made of the renewal moments.

Typical realizations of the operating time in navigational systems are characterized by the exponential distributions of the lifetime and the time of failures due to the property called the ”memoryless” property. Let define the exponential process where the distribution functions as

$$F(t)=\begin{cases} 1-e^{-\lambda t} & \text{for} \quad t>0 \\ 0 & \text{for} \quad t\le 0 \end{cases} \tag{8}$$

$$G(t)=\begin{cases} 1-e^{-\mu t} & \text{for} \quad t>0 \\ 0 & \text{for} \quad t\le 0 \end{cases} \tag{9}$$

where $\lambda$, $\mu$ are failure and renewal rates. When substituting (8) to (7) we obtain

$$A_{exp}(t)=1-F(t)+\int_0^t [1-F(t-x)]dH_\Phi(x)=e^{-\lambda t}+\int_0^t \left[1-\left(1-e^{-\lambda(t-x)}\right)\right]dH_\Phi(x) \tag{10}$$

where $A_{exp}(t)$ denotes the availability of the certain value of position error in the navigational system in the case of the exponential life and failure times distributions. After few simple transformations finally form could be find as

$$A_{exp}(t)=\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} \tag{11}$$

**4. DGPS Numerical example**

In order to evaluate new characteristics of the modernized DGPS system, a measurement campaign took place over the time span between 30th of June and 26th August 2009. Pseudorange (PRC) corrections type 9-3 broadcasted from DGPS reference station Rozewie were received and recorded by a single frequency (L1) receiver installed in Gdynia, 40 km from the transmitting MF beacon. These tests were focused on evaluation of horizontal and vertical accuracies of the 3D position. Results were based on long term observations (26 days) registered with 10s sampling rate. Data files were containing position time series in the format of NMEA 0183 standard, GGA referenced to WGS-84 datum ( $a = 6378137.00\,\mathrm{m}$, $b = 6356752.314\,\mathrm{m}$). The reference point for he receiver was set in the Port of Gdynia with coordinates: 54 31.75524 N, 18 33.57418 E, H = 68.07 m. The previous one (in 2005) was installed almost at the same position. Obtained results (statistics) has been compared to ones performed in 2005.

Let's define an absolute accuracy as a the statistical difference between position measurements and a surveyed benchmark for any point within the service volume over a specified time interval. When there is no information about surveyed coordinates the statistics could be related to estimated (average) position (repeatable accuracy).Measured ellipsoidal coordinates were transformed to Gauss-Kruger (X,Y) conformal coordinates and presented below:

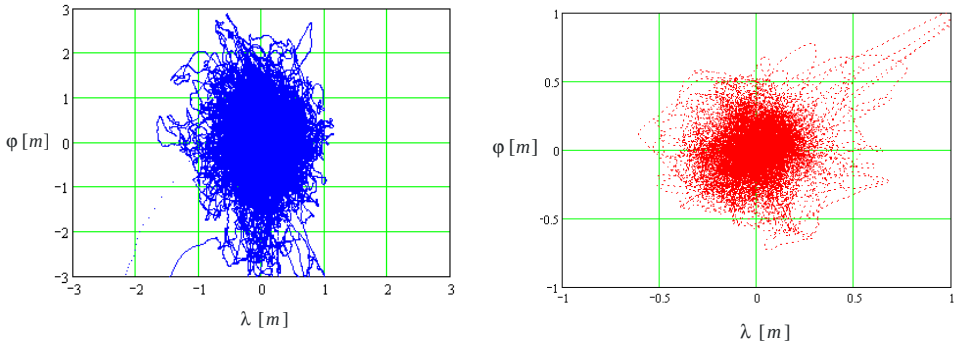


*Fig. 3. Scatter plots relative to average position (left-previous, 2005, right- current, 2009)*

Availability functions and their limited values are presented below. The functions were calculated for 3 decision value : 0.5, 1.0 and 1.5 m. The same analyses were done for both campaign (2005 and 2009).

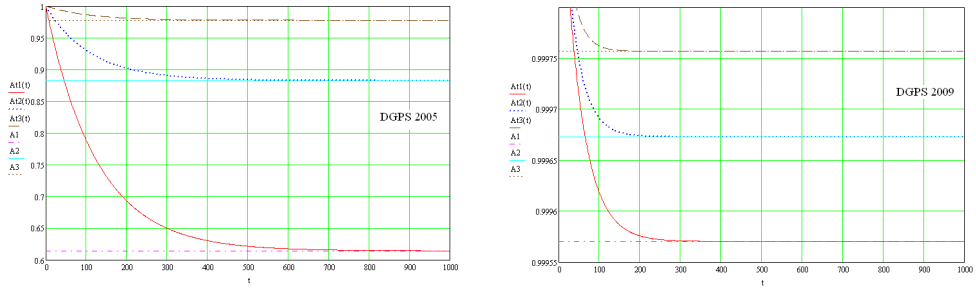


*Fig. 4. Availability functions:* $At1(t)$ for $U = 1\,m$, $At2(t)$ for $U = 1.5\,m$ $At3(t)$ for $U = 2\,m$ *and its limited values:* $At1, At2, At3$ *for DGPS system in 2005 (left) and 2009 (right)*

Presented functions (Fig. 4) have shown significant improvement of the accuracy of DGPS positioning in 2009 calculated based on the presented availability model.

## 5. CONCLUSIONS

The article presents the mathematical model of availability of the certain value of position error calculation. The classic approach to analogous estimation is characterized by the following assumptions: the determining error is a random variable, it does not respect playtimes of the system work and new one suggested approach treats the lifetimes and the times of failure as the random variables being in relation with the fixed value of the position error. Presented method allowed to compare the accuracy improvement after modernisation of the Polish DGPS.

## 6. REFERENCES

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