viscoelasticity, time-domain viscoelasticity, frequency-domain viscoelasticity, rheological model, generalized maxwell model, prony series expansion, asphalt concrete, complex modulus, flexible pavement design

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IMPLEMENTATION OF GENERALIZED VISCOELASTIC MATERIAL MODEL IN ABAQUS CODE

An accurate description of behavior of bituminous mixes is necessary to adequately predict and evaluate the time dependent characteristics and the evolution of pavement distress. The generalized Maxwell model, currently considered to be one of the most suitable to characterize mechanical behavior of viscoelastic materials and widely used in many commercial FEM codes, is presented. A Prony series expansion implemented in ABAQUS code, is used to express the material's behavior. The theoretical background of the model is briefly discussed. Finally, the model was efficiently used for numerical simulation of stress relaxation experiment.

IMPLEMENTACJA NUMERYCZNA UOGÓLNIONEGO MODELU MATERIAŁU LEPKOSPRĘŻYSTEGO W ŚRODOWISKU MES ABAQUS

W procesie projektowania nawierzchni podatnych, opartym na metodzie machanistycznej, kluczową rolę odgrywa określenie stanów odkształceń i naprężeń w konstrukcji drogi. Coraz częściej stosowanym do tego celu narzędziem jest metoda elementów skończonych, umożliwiająca uwzględnienie niesprężystych właściwości mieszanek mineralno asfaltowych. W pracy przedstawiono implementację uogólnionego modelu Maxwella w komercyjnym systemie MES ABAQUS. Pokrótce omówiono podstawy teoretyczne, a następnie pokazano, w jaki sposób na podstawie aproksymacji wyników badań doświadczalnych wyznaczane są parametry modelu reologicznego. Przeprowadzono symulację numeryczną procesu relaksacji naprężeń, a wyniki porównano z rozwiązaniem analitycznym dla modelu Burgersa.

1. INTRODUCTION

Flexible Pavements are multilayered systems constructed of bituminous and granular materials. Engineers involved in design of flexible pavements are continually looking for an effective analytical tool to assist in analyzing pavement structures. Such a tool will facilitate the establishment of a performance-based design, capable of extending the service life of roads. An ideal design tool consists of a structural model capable of predicting the state of stresses and strains within the pavement structure under the action of traffic and

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environmental loading. To carry out such analysis effectively, the design tool should be equipped with material models capable of capturing the mechanistic response of the various materials used to construct the road structure.

Since the early 1960s, pavement engineers have gone from empirical to mechanistic-empirical methods for pavement design. The latter is basically divided in two parts:

- a mechanistic part related to the prediction of stresses, strains and deflections in the pavement layers due to mechanical load on the pavement surface,
- an empirical part of the method relates the calculated structural response to cracking and permanent deformation by means of performance models.

Burmister's elastic layered theory [2], which assumes isotropic, homogenous and linear elastic material properties, is widely used for pavement analysis today. On the other hand it is well known that asphalt mixtures have a viscoelastic behavior [4]. Thus, its mechanical response exhibits time and rate dependency and the consideration of its behavior as elastic is not realistic. Structural pavement responses, such as stresses and strains, can be more accurately predicted by the consideration of the viscoelastic nature of the asphalt mixture. Though the concept of viscoelasticity is old in origin, its application to pavement design and analysis has not been dominant till now.

Early mechanics based research studies mainly focused on the viscous properties of asphalt concrete. In his series of papers, Van der Poel [9] advocated a linear viscoelastic model for asphalt. Subsequent research efforts concentrated on developing various viscoelastic models for asphalt concrete, e.g. Monismith [7], Kim[6], Judycki [5].

In comparison with the mechanical-empirical design based of theory of elasticity, the most important characteristic of the viscoelastic design is the recognition and application of viscoelastic behavior of hot-mixes-asphalt (HMA) materials in mechanical analysis. The inclusion of a viscoelastic constitutive model into a Finite Element (FE) program to simulate pavement response for pavement structures is getting more dominant, which will overcome inherent shortcomings of the elastic layered theory that was extensively used in the elastic design stage.

In order to develop improved knowledge of the actual behavior of flexible pavements subjected to moving load different rheological models composed of various combinations of springs and dashpots can be utilised. During the last decades several rheological models have been proposed in order to describe the mechanical behavior of bituminous mixtures. However, many of those models are only capable of representing the material in terms of stiffness and phase angle within a limited frequencies range.

Burgers model is one of the simplest models capable of representing a bituminous material in terms of stiffness and phase angle within limited frequency ranges. However if a wider range is to be covered, more advanced models are required. In this respect, the generalized Maxwell model and Huet-Sayegh model have proven to be the currently most suitable models available to characterize mechanical behavior of viscoelastic materials.

Burgers model and Huet-Sayeh model are included in VEROAD, a viscoelastic response program, while generalized Maxwell model is implemented in many commercial Finite Element Systems, e.g. Abaqus, Ansys. In this research, the generalized Maxwell model was selected to represent the behavior of HMA materials which are mechanically approximated by a Prony series. Implementation of the model in ABAQUS Version 6.8 [1] has been examined.

2. PRONY SERIES REPRESENTATION OF LINEAR VISCOELASTICITY

2.1 Constitutive relations of the viscoelastic material

The uniaxial, nonaging, isothermal stress-strain equation for a linear viscoelastic material can be represented by a Boltzmann superposition integral,

$$\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau \tag{1}$$

where $E(t-\tau)$ is the time-dependent “relaxation modulus” that characterizes the material’s response. This modulus can be drown from rheological models composed of various combinations of elastic elements (springs) and viscous elements (dashpots).

The simplest mechanical model imitating linear viscoelastic behavior is one spring combined in series with one dashpot called the Maxwell model. The behavior of this element, where a spring and a dashpot are connected in series, is expressed as:

$$\varepsilon(t) = \frac{\sigma(t)}{E} + \frac{\sigma(t)}{\eta} \tag{2}$$

where: E - the elastic modulus

h- the viscosity parameter

If a constant strain ε_0 is suddenly applied at *t=0* and held at that value for subsequent times, we can show that, for *t>0*, the solution of the equation (2) is given by

$$\sigma(t) = E\varepsilon_0 \exp\left(\frac{-t}{\tau_M}\right) \quad , \quad \tau_M = \frac{\eta}{E} \tag{3}$$

which implies that on start-up of shear, the stress growth is delayed. Hence the stress relaxes exponentially from its equilibrium value to zero. The rate constant τ_M is called the 'relaxation time'.

Maxwell model is generally too over simplified to describe real linear viscoelastic behavior. For example it can not simulate creep behavior well. What is more a real material does not relax with a single relaxation time as predicted by the previous models.

To overcome these problems, generalized model, shown in Fig.1, consisting of a spring and multiple Maxwell elements in parallel can be used. Then the total stress is given by

$$\sigma(t) = E_\infty\varepsilon_0 + \sum_{i=1}^{n} E_i\varepsilon_0 \exp\left(\frac{-t}{\tau_i}\right) = \varepsilon_0\left[E_\infty + \sum_{i=1}^{n} E_i \exp\left(\frac{-t}{\tau_i}\right)\right] \tag{4}$$

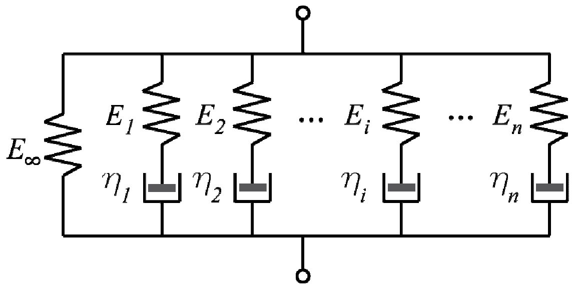

Fig.1. Schematic of generalized Maxwell model consisting of n Maxwell elements connected in parallel. In each Maxwell element, a spring (E_i) and dashpot (h_i) are connected in series. An extra isolated spring is added in parallel to represent the final (or equilibrium) modulus (E_∞).

The relaxation modulus *E(t)* is defined as

$$E(t) = \frac{\sigma(t)}{\varepsilon_0} = E_\infty + \sum_{i=1}^{n} E_i \exp\left(\frac{-t}{\tau_i}\right) \tag{5}$$

Which is essentially the Prony series representation. Here E_∞ is the final (or equilibrium) modulus, and $E_0 = E_\infty + \sum_{i=1}^{n} E_i$ is the instantaneous modulus. A pair of E_i and τ_M is referred to as a Prony pair. Mathematically, equation (5) can be rewritten as

$$E(t) = E_0 - \sum_{i=1}^{n} E_i \left(1 - \exp\left(\frac{-t}{\tau_i}\right)\right) \tag{6}$$

2.2 Material characterization

Typically, two types of testing methods are used to characterize the viscoelastic properties of asphalt concrete: dynamic modulus testing and constant static testing. In the first case, the material is characterized by the complex module *(E*)* and the phase angle (f). Under constant static load, the viscoelastic behavior is characterized through the creep compliance or the relaxation modulus. In this research, dynamic modulus testing was chosen as the measure of the complex modulus of HMA materials. Values presented in Tab.1 were extracted from literature [3]. The material is characterized by the phase angle *(f)* and the complex modulus *(E*)* which can be expressed as storage modulus and loss modulus using the following relation (7)

$$E^* = E' \cos(\varphi) + E'' i \sin(\varphi) \tag{7}$$

Tab. 1. Experimental results of complex modulus for various frequencies of excitation

Lp.	Frequency f [Hz]	Storage modulus *E'* [MPa]	Loss modulus *E''* [MPa]
1	0,5	742	1187
2	1	1089	1677
3	2	2046	2194
4	5	3072	2864
5	10	4728	3694
6	20	7047	4753
7	35	7844	4807

2.3 Numerical implementation

In many modern Finite Elements Systems (Abaqus, Ansys), viscoelasticity is implemented through the use of Prony series. The shear and volumetric responses are separated, and the well-known relationships between shear modulus G and bulk modulus K are shown below:

$$G = \frac{E}{2(1+\nu)} \quad , \quad K = \frac{E}{3(1-2\nu)} \tag{8}$$

Instead of having constant values for G and K (and by extension, elastic modulus E and Poisson's ratio n), these are represented by Prony series expansion of the dimensionless relaxation modulus. For simplicity, only the shear term G will be considered for subsequent discussions. Dividing both sides of the equation for relaxation modulus by instantaneous modulus G_0 yields:

$$g_R(t) = 1 - \sum_{i=1}^{n} g_k \left(1 - \exp\left(\frac{-t}{\tau_i}\right)\right), \quad g_R(t) = \frac{G(t)}{G_0} \tag{9}$$

Substitution in the small-strain expression for shear stress yields:

$$\tau(t) = G_0 \left(\varepsilon^{DEV} - \sum_{i=1}^{n} \frac{g_i}{\tau_i^G} \int_0^t \exp\left(\frac{-s}{\tau_i^G}\right) \varepsilon^{DEV}(t-s)ds \right) \tag{10}$$

The parameters necessary to fully model the time-dependent volumetric and shear behavior can be defined in one of four ways: direct specification of the Prony series parameters (k_i, g_i, $\tau_i^K = \tau_i^G = \tau_i$), inclusion of creep test data, inclusion of relaxation test data, or inclusion of frequency-dependent data obtained from sinusoidal oscillation experiment.

In this research the nonlinear curve-fitting utility of commercial FEM software ABAQUS was employed to determine Prony pairs, which can also be used as input for subsequent FEM stress simulations.

Prony series relaxation functions can be related to the storage and loss moduli using analytical expressions (11). These expressions are used in a nonlinear least-square fit to determine parameters.

$$G_s(\omega) = G_0\left[1-\sum_{i=1}^{n} g_i\right] + G_0\sum_{i=1}^{n}\frac{g_i\tau_i^2\omega^2}{1+\tau_i^2\omega^2}\ , \quad G_l(\omega) = G_0\sum_{i=1}^{n}\frac{g_i\tau_i\,\omega}{1+\tau_i^2\omega^2} \tag{11}$$

where: $G_s(\omega)$ - the storage modulus

$G_l(\omega)$ - the loss modulus

Let's consider nondimensional shear relaxation function $g(t) = G(t)/G_\infty - 1$ and its Fourier transform $g^*(\omega)$. The real and imaginary part of the function $\omega g^*(\omega)$, which can be expressed by equations (12), are listed in Tab.2.

$$\omega\Re(g^*) = \frac{G_l}{G_\infty} \quad , \qquad \omega\Im(g^*) = 1 - \frac{G_s}{G_\infty} \tag{12}$$

By using the procedure of Prony series calibration for viscoelastic material, 6 unknowns of relaxation time and stiffness modulus were obtained, which are listed in Tab.3. A fixed value of Poisson's ratio of 0.3 was used in the analysis.

Tab. 2. Frequency-dependent test data

Lp.	**Frequency f [Hz]**	**wg* Real**	**wg* Imaginary**
1	0,5	1.18727	0.25811
2	1	1.67734	-0.08927
3	2	2.1941	-1.04599
4	5	2.8644	-2.07169
5	10	3.6940	-3.72807
6	20	4.7531	-6.04682
7	35	4.8070	-6.84429

Tab. 3. Prony series parameters of the asphalt concrete used in the study

Lp.	τ_i [s]	g_i	$G_i = g_i G_0$ **[Mpa]**
1	5.5318 e-4	6.7008 e-1	7245
2	8.612 e-3	2.0422 e-1	2208
3	6.1742 e-2	0.9013 e-1	974

2.4 Numerical example

A simple FEM simulation was performed with the time-dependent material properties tabulated in Tab.3 to cross-check the material's behavior before this material property is implemented in more complicated models. Instantaneous shear strain was applied to a one-element specimen and held for up to 10s to simulate stress relaxation experiment. The stress response was divided by the constant strain to obtain the relaxation modulus, E(t), which is presented in figure 2.

Relaxation curve obtained from analytical and numerical solutions for generalized Maxwell model was compared with the curve predicted by Burgers model. The four parameters representing the stiffness modulus were also obtained from [3]. This simplest model was able to describe the viscoelastic behavior in reasonable way, although there was significant difference in instantaneous and long-term moduli. The comparison is shown in figure 3.

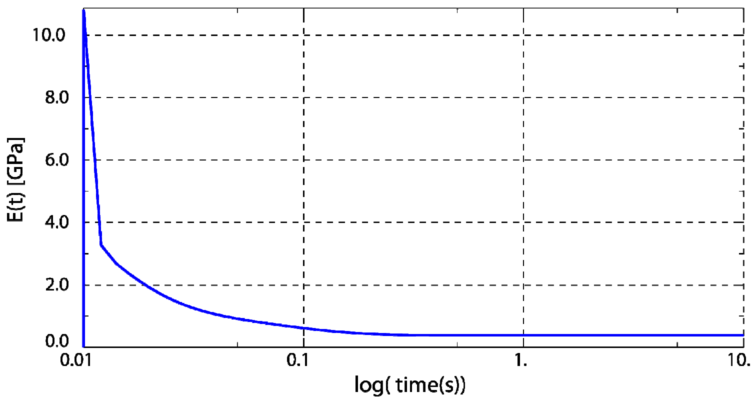

Fig.2. Verification of the viscoelastic property of a molding compound with a FEM model. The relaxation moduli defined by the Prony pairs.

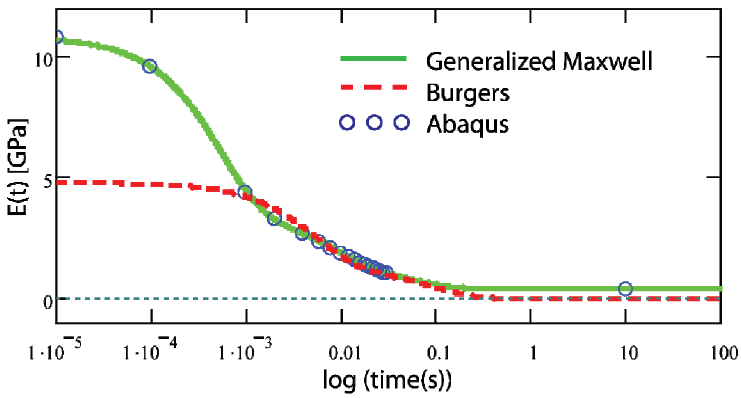

Fig.3. Comparison between analytical solution for generalized Maxwell model, numerical integration obtained by Abaqus FEM system and Burgers model.

3. CONCLUSIONS

An accurate description of behavior of bituminous mixes is necessary to adequately predict and evaluate the time dependent characteristics and the evolution of pavement distress. Finite element method is increasingly used in pavement simulations, because it provides greater flexibility in modeling boundary conditions, constitutive characteristics of materials and has other advantages over layered elastic solution.

The generalized Maxwell model is currently considered to be one of the most suitable to characterize mechanical behavior of viscoelastic materials. It is widely used in many commercial FEM codes. In this study implementation, known as Prony series, in ABAQUS code was examined. The theoretical background of the model was briefly discussed. Furthermore, this paper shows how to determine a time-domain Prony series representation from the complex modulus in the frequency domain. Finally, the model was efficiently used for numerical simulation of stress relaxation experiment.

Considering its accuracy and computation efficiency, presented model is suitable for flexible pavement structural analysis. However, it is known that viscoelasticity does not contribute significantly towards rutting of the pavement, so future work should be focused on introduction of permanent deformations into the model.

4. REFERENCES

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