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THE UTILISATON OF ADVANCED VEHICLE DYNAMICAL MODEL IN OPTIMISING THE SHAPE OF RAILWAY TRANSITION CURVES

Abstract: This paper describes the concept and method by present authors of searching for proper shape of transition curves. In the concept and the method the advanced dynamical model of vehicle-track system, capable of simulation, and mathematically understood optimisation methods are exploited. Principles and the most important details of the method are obviously represented in the paper. They are supplemented with description of the software built to solve the issue of transition curves formation (optimisation) effectively. At the end results generated by this software are presented quite comprehensively.

Keywords: optimisation, numerical simulation, transition curves, railway vehicle dynamics, track-vehicle interactions

1. INTRODUCTION

The problem discussed in present article is the search for proper shape of railway transition curves. It is going to be done with regard to advanced vehicle model, dynamical track-vehicle and vehicle-passenger interactions, and optimisation methods. The search for proper shape means to the authors evaluation of the curve features based on chosen dynamical quantities as well as the generation of such shape with use of mathematically understood optimisation methods. The studies performed so far and those to be done in the future have got a character of the numerical tests. The key element for these tests is use of numerical simulation and the corresponding simulation software built by the authors. This software enable to simulate dynamics of the vehicle-track system exploiting complete dynamical model of the railway vehicle. Results of the simulations can be utilised in two general ways. First is the direct use to get the complete knowledge about the dynamical interactions and behaviour of the vehicle in transition curves. Second is the use of simulation results to determine the objective function in the process of optimising the

shape of transition curve (TC). This second way of the results utilisation is discussed in the paper first of all.

1.1. Short literature survey

Review of the literature given below cannot be complete in terms of the cited and discussed publications since it would need a separate paper. Such a paper by present authors is [1]. Below we discuss the literature jointly, trying to focus on the features being the most common but not necessarily those most modern ones. Next, the criteria of TCs evaluation are classified, which are used most often.

In opinion of present authors the most general classification of the works on TCs might be their division into two groups. The first group includes those works that originate in the issues of railway road design (construction). So, these works deal with the railway infrastructure or in a wider view with the civil engineering. The second group is oriented towards railway vehicle dynamics. Thus strong relation to the mechanical engineering is clear. It seems that increase in total number of works concerning TCs that happened lately [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] did not change a proportion between number of the works in both groups. Despite increase in number of works in the second group [11, 12, 13, 14, 15, 16, 17], the advantage of works' number in the first group is still a fact.

Recently increase in number of publications that deal with transition curves, both the railway and the road ones, can be noticed. Also some qualitative change in their content can be observed. It consists in attempts to diverge from the standard and to look for new, more modern methods of evaluating properties of TCs. Despite these, some earlier visible limitations of those works still exist, in present authors opinion. Namely, the analysis is rather rare which takes account of advanced dynamics of whole vehicle-track system. Present authors do not know method applied in practice (approved as a design tool), which use complete dynamical model of vehicle in formation of railway TCs. Many of the methods in use have got the traditional character. They are based on the traditional criteria (discussed below) and often refer to very simple vehicle model. The authors failed to find publications that exploit directly mathematically understood optimisation methods in formation of TCs, basing on objective functions calculated as a result of the numerical simulations. There are some works where selected quantities of interest, rather than the shape of the curve itself, are optimised instead.

In many works from the first group approach to the track-vehicle interactions is traditional, e.g. [18]. It is limited to discussing the vehicles jointly and studying the selected effects (quantities) in the car body. In such works the fundamental criteria of 3-dimensional TCs' formation are still crucial. Namely, the physical quantities that characterise effects on the passenger and eventually on the cargo should not exceed values that are acknowledged as acceptable [14, 18]. The corresponding relations refer to: unbalanced lateral acceleration $a \leq a_{lim}$, velocity of the a change $\psi \leq \psi_{lim}$, and velocity of wheel vertical rise along the superelevation ramp $f \leq f_{lim}$. Some up-to-date works extend these criteria with additional quantities and search for their courses. Such a quantity is the second derivative of a with respect to time t or l, in case of constant v. As to the courses (of the a first and second derivatives most often), the continuity (no abrupt change in values), differentiability (no bends) and so on are demanded. Despite that extension, such criteria

do not take account of the dynamical properties of particular vehicle, including track-vehicle interactions in particular conditions, or effects on vehicle bogie. These are different than those adopted in the discussed criteria, where track has infinite stiffness and no geometrical irregularities and vehicle is represented by a single rigid body or a particle.

Worthy of discussion are the criteria in the literature for evaluating TCs properties. Let us classify them into tree groups called: geometrical, geodetic, and dynamic criteria. In case of the geometrical criteria the appropriate boundary conditions at TC terminal points must be satisfied [14, 18, 19]. Also so called consistency conditions inside TC have to be satisfied. They ensure consistency (identity) of curvature k, superelevation ramp h, and unbalanced acceleration a functions. Apart from the acceptable boundary and maximum values also shape of the $(da/dt)=\psi$ and sometimes d^2a/dt^2 courses is of interest, e.g. [18].

The geodetic criteria compare different types of TCs from point of view of ease of their setting down and maintenance during operation, e.g. [11, 20].

In case of the dynamical criteria [11, 12, 13, 14, 15, 16, 17] dynamical response of the system on the input represented by TC (eventually including superelevation ramp) is of interest. Two approaches can be distinguished here. The first is to build relatively simple vehicle model that represents all vehicles and next to solve it analytically or numerically, e.g. [13, 16, 20]. The second is to build advanced vehicle or vehicle-track model and to solve it numerically (simulation) and then to analyse the results, e.g. [11, 12, 14, 15]. This type of criteria and works in type of the last mentioned confirm us in a conviction that the approach represented in our paper should be found justified and the results interesting.

2. AIMS AND SCOPE OF THE ARTICLE

2.1. The aim and scope in general

The main aim of the studies represented by present article is to elaborate and test the numerical method of TCs form optimisation. This is in agreement with the general concept of evaluating and forming TCs formulated in Sec. 1.

As to the scope, several introductory assumptions (expectations) have to be mentioned. First, pure mathematical methods of optimisation are going to be used. Second, dynamical quantities being the result of simulation of railway vehicle advanced (complete) model will be used to determine the objective functions in the optimisation process. Third, possibility of calculation a dozen or so different objective functions should be ensured in the software. Forth, search for optimum solutions with objective functions based on traditional criteria should be secured. Fifth, possibility to adopt all fundamental demands concerning TC should be preserved in the software. Sixth, two-axle vehicle model as well as vertically and laterally flexible track models are going to be used. Seventh, full non-linear geometry and forces in wheel/rail contact have to be taken into account. Eight, polynomial TC is going to be optimised. Ninth, the highest order of the polynomial should equal at least 12. Tenth, optimisation of the entrance and exit TCs as well as of both simultaneously, for the compound route, should be possible.

2.2. The object, its model, and the corresponding software

In order to make the analysis easier and more clear relatively simple object, and hence its relatively simple model were utilised. To make direct reference to the earlier results possible, the same model of the system was used as in the earlier studies by present authors [14]. Consequently, all simulation studies were done for discrete vehicle model of 2-axle freight car, as described in refs. [14, 21]. Its structure is shown in Fig. 1c. It is supplemented with discrete models of vertically and laterally flexible track shown in Figs. 1a and 1b, respectively. Parameters of the models are within average range of values for 2-axle freight cars and for track, too. Linearity of the vehicle suspension system was assumed. So, linear stiffness and damping elements in vehicle suspension were applied. The same concerned the track models. Here also linear stiffness and damping elements were utilised. One can find all parameters of the models in [14, 21].

Vehicle model is equipped with a pair of wheel/rail profiles that corresponds to the real ones. That is a pair of the nominal (i.e. unworn) S1002/UIC60 profiles that are used all over the Europe. Non-linear geometry of this pair is introduced into the model in a form of the table with contact parameters. In order to calculate non-linear tangential contact forces between wheel and rail well known FASTSIM program by J.J. Kalker was applied. Normal forces in the contact are not constant but influenced by both the geometry and the dynamical effects influencing value of a wheelset vertical load.

Generalised approach to the modelling was used, as explained in [21, 22, 23]. Basically, dynamics of relative motion is used in that approach. This means that description of motion (dynamics) is relative to track-based moving reference frames. Dynamical equations of motion are equations of relative motion with terms depending on motion of the reference frames explicitly recorded. None of such terms is omitted in the equations. According to this method, the kinematic type non-linearities arising from rotational motions of bodies within our MBS model are taken into account, too. The term generalised refers fist of all to the generalised conditions of motion. So, the same generalised vehicle model describes vehicle dynamics in any conditions, i.e. in straight track (ST), circular curve (CC), and TC sections. The routes composed of such sections can also be analysed.

The route (section) of interest is characterised in the method by shape of the track centre line which is the general space (3-dimensional) curve. In railway systems such 3-dimensional objects are TCs with their superelevation ramps. A necessary condition to apply the method is description of the curves (sections) by parametric equations, with the
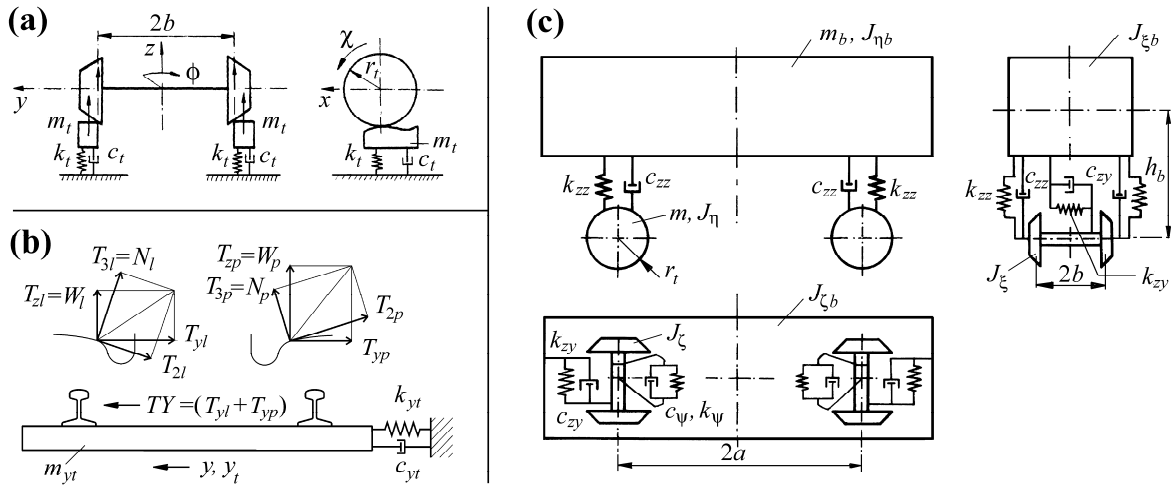
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Fig. 1. System's nominal model: (a) track vertically, (b) track laterally, (c) vehicle

curve's current length as the parameter. The cases of CC and ST are treated in the method as the special cases of 2-dimensional and 1-dimensional geometrical objects, respectively. Such an approach was described in [19, 23].

An important element in the method is description of kinematics of the track-based moving reference frames. Their motion comes out directly from the track centre line shape. The applied method of determination of the kinematical quantities on the basis of the parametric equations is presented most recently in [23].

3. METHOD FOR OPTIMUM TRANSITION CURVE FORMATION

3.1. The simulation software

Simulation software reflects in numerical form all the features specified above for the mathematical model. So, the original as well as characteristic features of the model are represented in the software. Despite many such features, in general view our software might be defined as a typical railway vehicle dynamics simulation software. We mean that the aims for its use and results generated are those typical.

The software in view was built by the authors on their own, so it is not a commercial code. It is also not a software for automatic generation of equations of motion (AGEM) but the one corresponding to a particular mechanical system. Here it corresponds to the system defined in Fig. 1. It was developed, tested, and used in research for many years. The base in this software is set of the 2nd order ordinary differential equations (ODEs). These equations of motion were derived from Lagrange equations of type II adapted to description of relative motion. General form of such equations is presented in [19, 21]. Number of the equations equals a number of degrees of freedom (DOFs) of the system. Complete vehicle-track system possess 18 DOFs. The equations of 2nd order are transformed within the code into the 1st order ODEs to integrate them. The integration is done with use of methods capable to solve stiff differential equations (Gear's methods).

Direct result of the numerical integration are obviously the system co-ordinates and velocities. Apart from them, the broad set of other supplementary quantities can also be obtained. For example these could be forces and torques exploited in the simulations. The results can either be recorded in the resultant files or used directly in the calculations. Both the direct and the supplementary results are used while calculating values of objective functions during the optimisation.

3.2. Type of the transition curve and conditions for it

Type of a TC chosen for optimisation is the polynomial TC of any order $n \geq 4$. It is defined by Eqs. (1)-(4) that are related to space curve parametric equations:

$$ y = \frac{1}{R}\left(\frac{A_n l^n}{L^{n-2}} + \frac{A_{n-1} l^{n-1}}{L^{n-3}} + \frac{A_{n-2} l^{n-2}}{L^{n-4}} + \frac{A_{n-3} l^{n-3}}{L^{n-5}} + \ldots + \frac{A_4 l^4}{L^2} + \frac{A_3 l^3}{L^1} \right) \tag{1} $$

$$ k = \frac{d^2 y}{dl^2} = \frac{1}{R}\left(n(n-1)\frac{A_n l^{n-2}}{L^{n-2}} + (n-1)(n-2)\frac{A_{n-1} l^{n-3}}{L^{n-3}} + \ldots + 3 \cdot 2 \frac{A_3 l^1}{L^1} \right) \tag{2} $$

$$ h = H\left(n(n-1)\frac{A_n l^{n-2}}{L^{n-2}} + (n-1)(n-2)\frac{A_{n-1} l^{n-3}}{L^{n-3}} + \ldots + 4 \cdot 3 \frac{A_4 l^2}{L^2} + 3 \cdot 2 \frac{A_3 l^1}{L^1} \right) \tag{3} $$

$$ i = \frac{dh}{dl} = H\left(n(n-1)(n-2)\frac{A_n l^{n-3}}{L^{n-2}} + (n-1)(n-2)(n-3)\frac{A_{n-1} l^{n-4}}{L^{n-3}} \right.$$
$$\left. + \ldots + 5 \cdot 4 \cdot 3 \frac{A_5 l^2}{L^3} + 4 \cdot 3 \cdot 2 \frac{A_4 l^1}{L^2} + 3 \cdot 2 \cdot 1 \frac{A_3 l^0}{L^1} \right) \tag{4} $$

where y, k, h, and i define curve lateral co-ordinate, curvature, superelevation, and inclination of superelevation ramp, respectively. The R, H, L, and l define curve minimum radius (at its end), maximum susperelevation (at the curve end), total curve length, and current curve length, respectively. The A_i are polynomial coefficients ($i=n$, n-1,…., 4, 3) while n is polynomial order. Number of the polynomial terms (terms in Eqs. (1)-(4)) must not be smaller than 2. Definition of the curves as adopted here gives possibility of proper k and h values at TCs terminal points. They should equal 0 at the initial points and 1/R and H at the end points. Note, that values for both always equal 0 for l=0. In order to ensure 1/R and H values for l=L, normalisation of the coefficients is necessary. It is done with *WSR* coefficient defined in Eq. (5). The normalised coefficients A'_i are given in Eq. (6).

$$\left(n(n-1)A_n + (n-1)(n-2)A_{n-1} + \ldots + 4\cdot 3\cdot A_4 + 3\cdot 2\cdot A_3\right) = WSR$$

$$(1/WSR)\cdot\left(n(n-1)A_n + (n-1)(n-2)A_{n-1} + \ldots + 4\cdot 3\cdot A_4 + 3\cdot 2\cdot A_3\right) = 1$$

(5)

$$
\begin{aligned}
A'_n &= A_n/WSR & \ldots\ldots A'_5 &= A_5/WSR \\
A'_{n-1} &= A_{n-1}/WSR & A'_4 &= A_4/WSR \\
A'_{n-2} &= A_{n-2}/WSR \ldots\ldots & A'_3 &= A_3/WSR
\end{aligned}
$$

(6)

Finally, form of each TC being tested for optimum shape during the optimisation process is determined with the following parametric equations:

$$x \cong l$$

$$y = \frac{1}{R}\left(\frac{A'_n l^n}{L^{n-2}} + \frac{A'_{n-1}l^{n-1}}{L^{n-3}} + \frac{A'_{n-2}l^{n-2}}{L^{n-4}} + \frac{A'_{n-3}x^{n-3}}{L^{n-5}} + \ldots + \frac{A'_5 l^5}{L^3} + \frac{A'_4 l^4}{L^2}\right)$$

(7)

$$z = \frac{H}{2}\left(n(n-1)\frac{A_n l^{n-2}}{L^{n-2}} + (n-1)(n-2)\frac{A_{n-1}l^{n-3}}{L^{n-3}} + \ldots + 5\cdot 4\frac{A_5 l^3}{L^3} + 4\cdot 3\frac{A_4 l^2}{L^2}\right)$$

In order to ensure tangence of the *k* and *h* functions at their terminal points to the *k* and *h* functions for ST and CC, values of *i* function given in Eq. (4) should equal 0 for *l*=0 and *l*=L. Equation (4) always produce 0 for *l*=0 when its last term is removed. An this is the first condition that affects also rest of the equations. To ensure 0 value for *l*=L the round bracket in (4) with the last term omitted should equal 0. In order to make it true all the coefficients remain unchanged but not just one arbitrarily selected. Its value is taken as opposite to the sum of those unchanged. And this is the second condition, that affects also rest of the equations. In case the second coefficient is selected for change it would be:

$$\frac{1}{L}\left(n(n-1)(n-2)\frac{A'_n l^{n-3}}{L^{n-3}} + (n-1)(n-2)(n-3)\frac{A'_{n-1}l^{n-4}}{L^{n-4}} + \ldots + 4\cdot 3\cdot 2\frac{A'_4 l^1}{L^1}\right) = 0$$

$$A''_{n-1} = -\frac{1}{(n-1)(n-2)(n-3)}\left(n(n-1)(n-2)A'_n + \ldots + 5\cdot 4\cdot 3 A'_5 + 4\cdot 3\cdot 2 A'_4\right)$$

(8)

Some problem in optimising form of TC is a starting point (initial form of TC). In case of TC of 5th, 7th, and 9th orders we used the curves known in the literature as the best ones from point of view of the traditional criteria. For example, in case of the 5th order it was the Bloss (Glodner) curve. Generally, such curves ensure minimum of the centrifugal force integral inside a TC. This is satisfied for the simplest possible model, however, that is in fact represented by a particle. For the higher order polynomial TCs form of the initial TC was an arbitrary choice, possibly in connection with TCs found in the literature. The curves in view are given below.

$$y = \frac{1}{R}\left(-\frac{l^5}{10L^3} + \frac{l^4}{4L^2}\right) \tag{9}$$

$$y = \frac{1}{R}\left(\frac{l^7}{7L^5} - \frac{l^6}{2L^4} + \frac{l^5}{2L^3}\right) \tag{10}$$

$$y = \frac{1}{R}\left(-\frac{5l^9}{18L^7} + \frac{5l^8}{4L^6} - \frac{2l^7}{L^5} + \frac{7l^6}{6L^4}\right) \tag{11}$$

3.3. Optimisation method and objective functions

Let us formulate the optimisation problem that is going to be solved in the presented studies. So, one ought to find A'_i, being the polynomial coefficients, ensuring possibility of imposing constraints on their values. The constraints in the form $b_i < A'_i < c_i$ should enable to control form of the polynomials in case it is necessary. Namely, the forms of polynomials reasonable from practical point of view should be tested in succeeding steps of the optimisation process, rather than those being impossible to apply in a railway conditions. The problem just formulated is a classical formulation of the static constrained optimisation. It is realised with the library procedure that utilises moving penalty function algorithm combined with Powell's method of conjugate directions.

The difficulty of the problem solution lies in the very complex objective function (quality function). This function is calculated as a result of the numerical simulation of dynamical mechanical system (as described in Subsecs. 3.2 and 4.1). The main steps during calculation of the objective function are: generation of the new shape of TC, calculation of kinematical quantities (velocities and accelerations) that depend on this new shape, and solution of ODEs set.

At current stage of the studies nine quality functions (QF) were implemented in the software. They concern minimisation of: sum of lateral displacements for both wheelsets (QF_1); sum of squares of longitudinal and lateral creepages in wheel/rail contact for all wheels (QF_2); sum of yaw angles for both wheelsets (QF_3); wheel/rail wear represented by sum of products of tangential forces by creepages in longitudinal and lateral directions for all wheels (QF_4); wheel/rail wear represented by sum of products of normal forces and resultant creepages for all wheels (QF_5); sum of normal forces for all wheels (QF_6); sum of lateral accelerations of wheelsets and car body (QF_7); sum of lateral accelerations of wheelsets (QF_8); and lateral acceleration of car body (QF_9).

So far just three functions were tested. They are QF_2, QF_7, and QF_8. Their physical interpretation is as follows. Minimisation of QF_2 causes less wear of wheels and rails, of QF_7 reduces vehicle-track and vehicle-passenger interactions, while of QF_8 reduces vehicle-track interactions. These three QFs are given in Eqs. (12), (13), and (14):

$$QF_2 = L_C^{-1} \int_0^{L_C} \left(v_{xlp}{}^2 + v_{ylp}{}^2 + v_{xrp}{}^2 + v_{xrp}{}^2 + v_{xlk}{}^2 + v_{ylk}{}^2 + v_{xrk}{}^2 + v_{yrk}{}^2 \right) dl \qquad (12)$$

$$QF_7 = L_C^{-1} \int_0^{L_C} \left(|\ddot{y}_p| + |\ddot{y}_k| + |\ddot{y}_b| \right) dl \qquad (13)$$

$$QF_8 = L_C^{-1} \int_0^{L_C} \left(|\ddot{y}_p| + |\ddot{y}_k| \right) dl \qquad (14)$$

where L_C is distance where QF is calculated, v_x is longitudinal creepage, v_y is lateral creepage, and $\ddot{y}$ is lateral acceleration. Indices l, r, p, k, b refer to left hand-side, right hand-side, leading wheelset, trailing wheelset, and vehicle body, respectively.

3.4. General representation of the software

Scheme of the software is shown in Fig. 2. Two iteration loops are visible there. The first is an integration loop. This loop is stopped when distance l_{lim} being length of the route (usually the compound route ST, TC, and CC) is reached by vehicle model. The second is an optimisation process loop. It is stopped when number of iterations reaches limit value i_{lim}. This value means that i_{lim} simulations have to be performed in order to stop optimisation process. In the calculations done so far i_{lim} equal 100 was used. If optimum solution is reached earlier, i.e. for $i < i_{lim}$ then the optimisation process stops automatically and the corresponding results are recorded. The post-processor ensures record of A'_i coefficients in each optimisation step in the special file. Besides results of simulations for the initial and optimum TC shapes are recorded automatically in the resultant files.
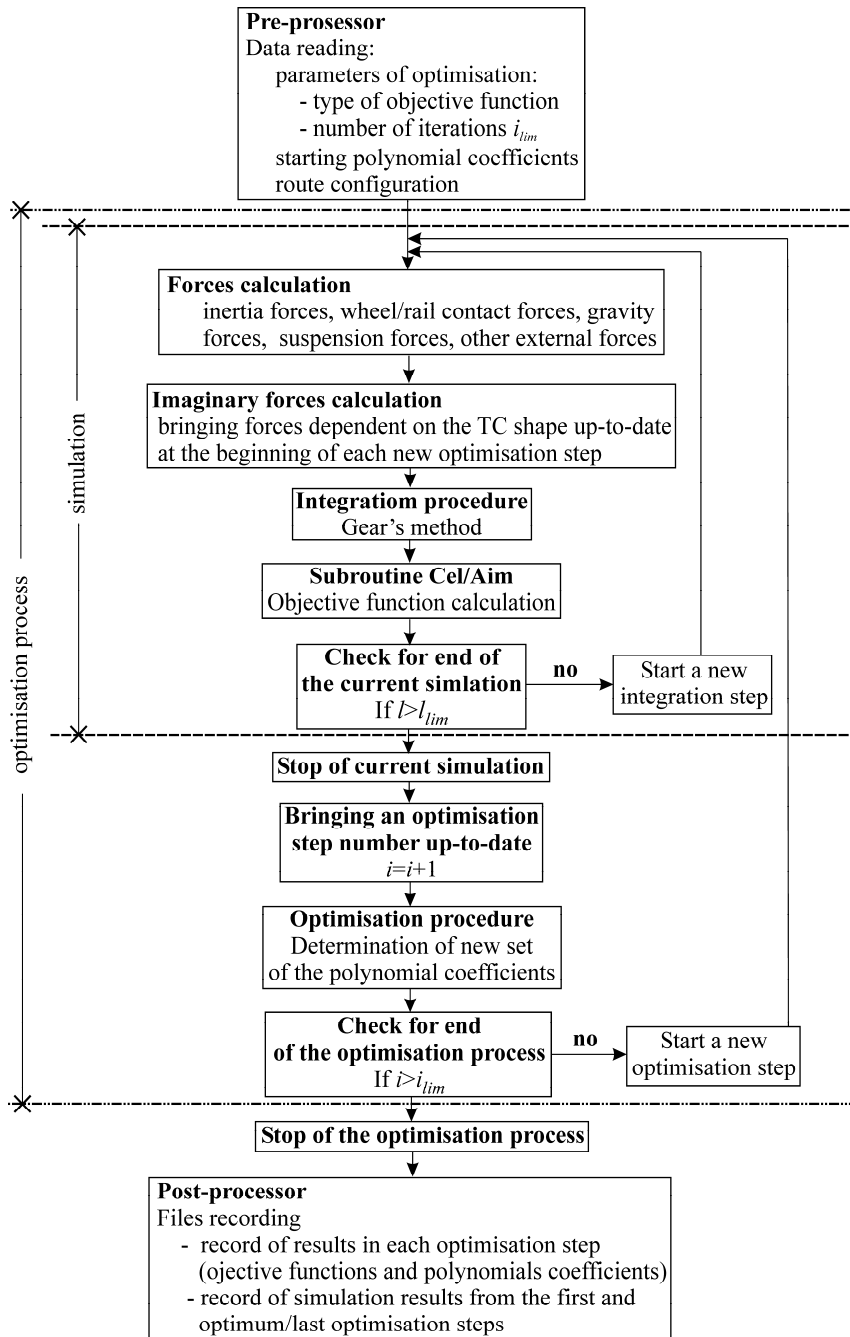
Fig. 2. General scheme of the optimisation software

4. RESULTS OF NUMERICAL TESTS

The results to be presented are divided into two parts. The aim for the first part is demonstration that the method works well and results of optimisations will be useful in the future analyses. The aim for the second part is to show what kind of results can be

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generated and how the first analyses look like. All the results below refer to the same compound route. Its data are: velocity v=45 m/s, (ST; L=50 m), (TC; L=102,5 m, R_{min}=600 m, H_{max}=0,16 m), (CC; L=100 m, R=600 m, H=0,16 m).

4.1. Results Aimed at testing the Software

It can be seen from Figs. 3 and 4 that searching for the optimum TC is successful. Usually quite wide range of the shapes is swept (tested). It can be seen in Figs. 3 and 4 that applied method of forcing the software to take proper values for k and h at TC's terminal points works right. By comparing these figures it can also be seen that the method ensuring tangence of k and h functions for TC at its terminal points to the functions for ST and CC works right, too. It can be observed for the shapes producing k and h values smaller than 0 or higher than $1/R$ and H that such values are trimmed off by the software to avoid model's "derailment". In case "derailment" persists, QFs are given very high values to stop interest of the software in such solutions. Coefficients A'_i for the optimised TCs are given in Sec. 6.
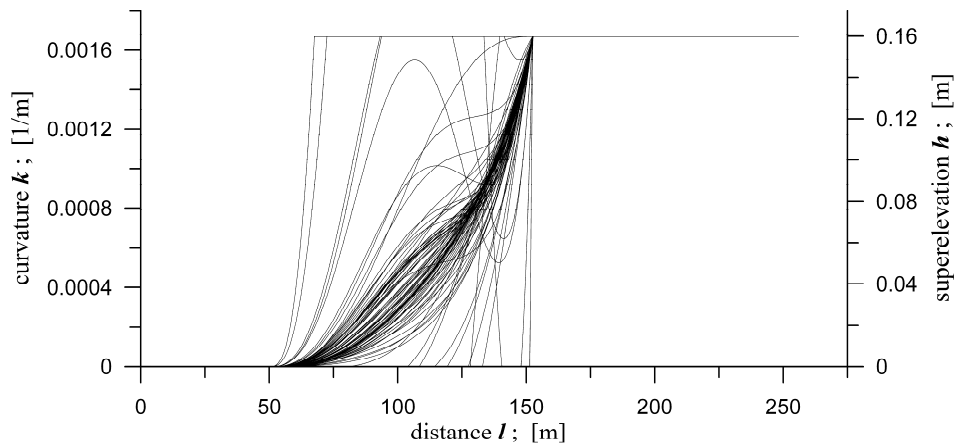

Fig. 3. Search for optimum 7th order TC's shape, represented by curvature k and superelevation h. No conditions for the tangence of the k and h is imposed. QF_2 is used
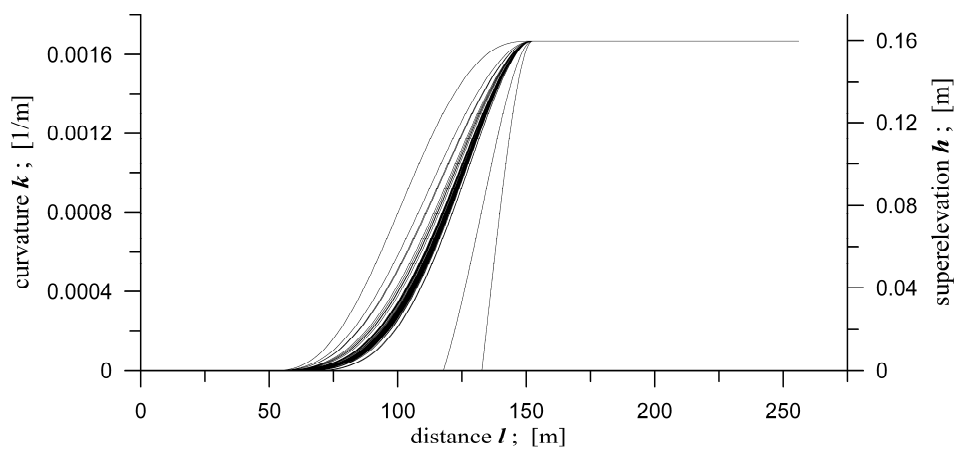

Fig. 4. Search for optimum 7th order TC's shape represented by curvature k and superelevation ramp. Condition for the tangence of the k and h is imposed. QF_2 is used

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4.2. Results of Optimisation and Dynamical Simulations

It can be seen in Figs. 5 and 6 that for different *QF*s optimisation process goes differently. Different sets of the curves are tested for being or not the optimum ones. Obviously different sets of A'_i coefficients for optimum shapes are obtained (see Sec. 6), too. Figure 7 represents optimised shape of the curves and features for such curves. Figure 7(a) represents the shapes in plane while Fig. 7(b) the shape of superelevation ramps. Basic features of the curves are shown in Figs. 7(b) and 7(c). Figure 7(c) shows curvatures and Fig 7(c) shows superelevation ramp slopes. Denotations used in these figures are OPF2, OPF7, and INIT for QF_2, QF_7 quality functions, and for the initial shape given with Eq. (11), respectively. Figs. 8 and 9 represent selected dynamical quantities that show improvement in the vehicle dynamical behaviour due to use of the optimised TC shape. Comparison for the initial (INIT) and the optimised (OPF7) curves is done there. Despite relatively small differences in these curves shapes and in their features seen in Fig. 7, the improvements revealed in Figs. 8 and 9, especially for the car body, are spectacular. Differences for the wheelsets also exist but are much smaller.
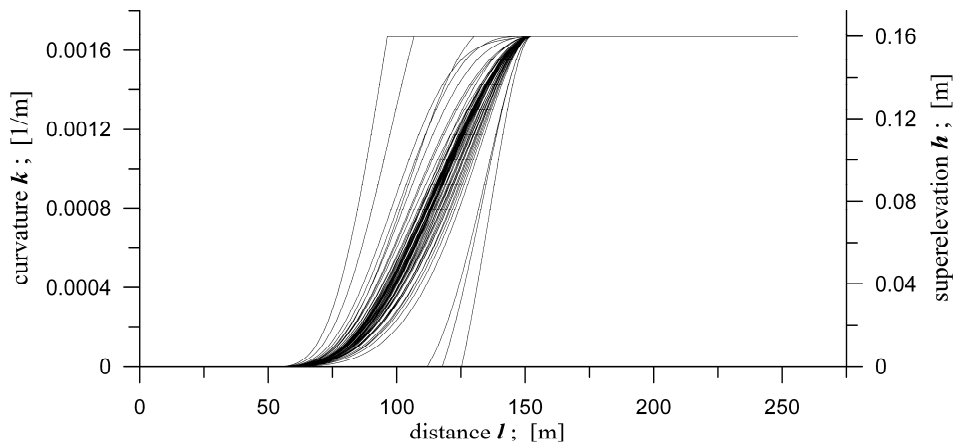

Fig. 5. Search for optimum 9th order TC's shape represented by curvature *k* and superelevation ramp. Condition for the tangence of the *k* and *h* is imposed. QF_2 is used.
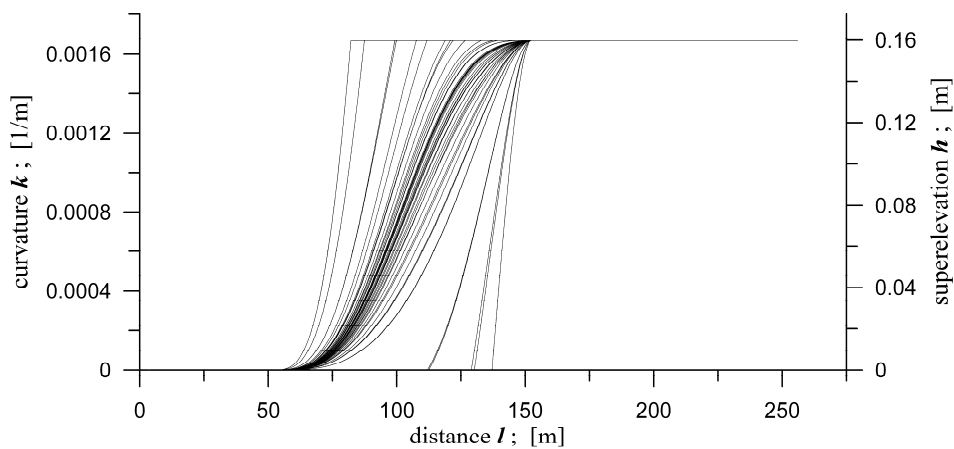

Fig. 6. Search for optimum 7th order TC's shape represented by curvature *k* and superelevation ramp. Conditions for the tangence of the *k* and *h* is imposed. QF_7 is used.
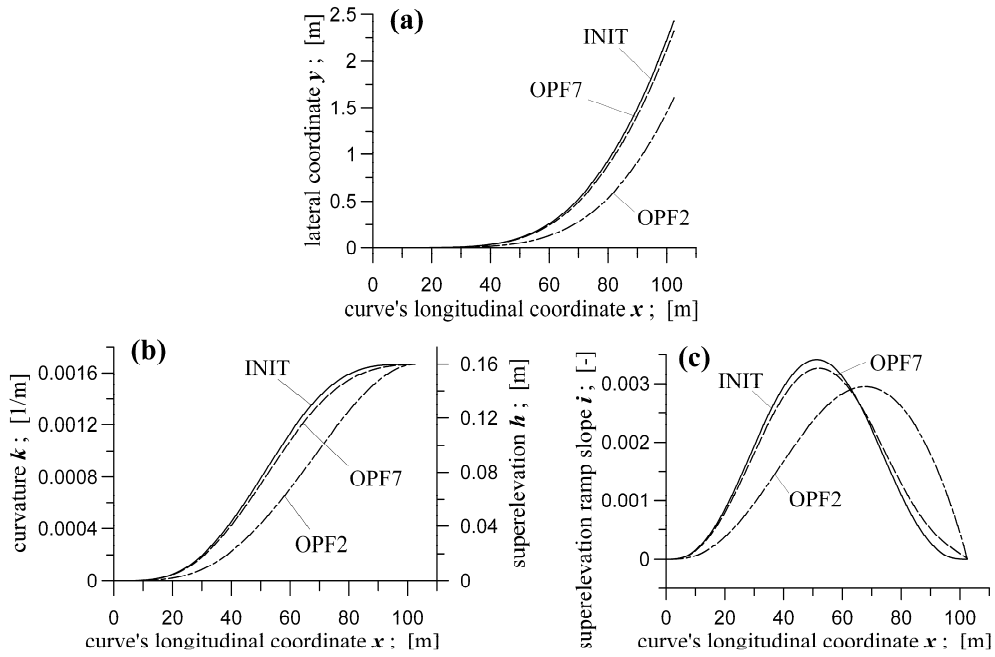
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Fig.7. Results of optimisation for QF_2 and QF_7 compared to the initial curve: (a) curve shapes, (b) curvatures and supelevations, (c) superelevation ramp slopes
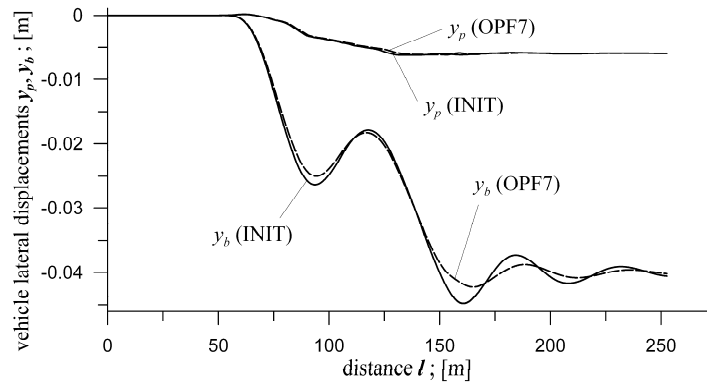

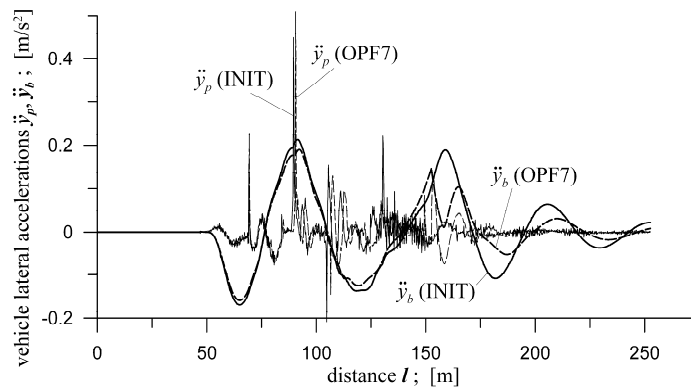Fig. 8. Vehicle model lateral displacements for optimised (QF_7) and initial TCs

Fig. 9. Vehicle model lateral accelerations for optimised (QF_7) and initial TCs

5. CONCLUSION

The idea and the corresponding software presented in current paper appeared to be effective tool in optimising polynomial TCs form. It proved true for different quality functions based on the dynamical quantities from the advanced vehicle-track system. It was possible to generate A'_i polynomial coefficients in all case studies done so far. The sets of coefficients corresponding to TCs optimised in present paper, recorded from the highest to the lowest order of the terms, are as follows: Fig. 3, {0.19999, -0.49975, 0.37363}; Fig. 4, {1.3015, 2.4063, –0.26081, 0.0081477}; Fig. 5, {-0.26498, 1,3646, -2.736, 2.0801}; Fig. 6, {-0.28031, 1.25, -1.9939, 1.1667}. Number of the terms depends on the data taken before each optimisation. To get the TCs' equations, the above coefficients have to be introduced into Eq. (7).

It was shown that TCs properties can be improved by omission the lowest order terms. The example are superelevation slopes (inclinations) in Fig. 7c. The TC matching OFP2 curve has one term less than TCs matching OFP7 and INIT curves.

In the foreseeable future the following issues deserve our attention. More *QF*s should be tested and proposed. Some could take account of the several factors with use of the weighting factors, some could be calculated for the selected parts of TCs. Next, optimisations for the exit TCs and for the routes that include both the entrance and exit TCs would be desirable. Our software is capable of doing these. Number of the case studies necessary to perform in order to recommend any optimum TC shape for practice is really large.

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WYKORZYSTANIE ZAAWANSOWANEGO MODELU DYNAMICZNEGO POJAZDU W OPTYMALIZACJI KSZTAŁTU KOLEJOWYCH KRZYWYCH PRZEJŚCIOWYCH

Streszczenie: Niniejszy artykuł opisuje koncepcję i metodę poszukiwania właściwego kształtu kolejowych krzywych przejściowych. W koncepcji oraz metodzie wykorzystuje się zaawansowany model dynamiczny układu tor-pojazd, z możliwością symulacji, oraz matematycznie rozumiane metody optymalizacji. Oczywiście w artykule przedstawiono podstawy jak i najważniejsze szczegóły metody. Są one uzupełnione o opis oprogramowania zbudowanego dla efektywnego rozwiązania zagadnienia kształtowania (optymalizacji) krzywych przejściowych. Na koniec, całkiem obszernie przedstawione zostały wyniki generowane przez oprogramowanie.

Słowa kluczowe: optymalizacja, symulacja numeryczna, krzywe przejściowe, dynamika pojazdów szynowych, oddziaływania tor-pojazd