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genetic algorithm, individuals, chromosomes, population, selection, crossover, mutation, modal amplitudes, multi-objective optimization, multi-objective function, design variables, Pareto-optimal solution

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APPLICATION OF GENETIC ALGORITHMS IN MULTI-OBJECTIVE OPTIMIZATION IN ROOM ACOUSTICS

In the paper, a problem of proper, optimum distribution of an acoustic absorption materials on the room's boundaries, to obtain desirable acoustic pressure level has been presented. Acoustic pressure distribution inside of a room can be described using modal analysis assumptions. Multi-objective function was created applying room's frequency response function and costs of absorption material distribution. Impedance values on each boundary were chosen as design variables. Search of the criteria minimum of the objective function, using genetic algorithm, has been conducted. As the result Pareto optimal solution i.e. set of material with the specific normal-absorption coefficient, properly distributed on boundaries has been found.

WYKORZYSTANIE ALGORYTMÓW GENETYCZNYCH DO OPTYMALIZACJI WIELOKRYTERIALNEJ W AKUSTYCE POMIESZCZEŃ

Zaprezentowano problem optymalizacji rozmieszczenia materiału absorbującego akustycznie, na brzegach pomieszczenia zamkniętego. Do opisu rozkładu ciśnienia akustycznego w pomieszczeniu zastosowano analizę modalną. Zdefiniowano wielokryterialną funkcję celu wykorzystując odpowiedź częstotliwościową pomieszczenia oraz funkcję kosztów rozmieszczenia materiału absorpcyjnego. Wartości impedancji na poszczególnych brzegach pomieszczenia zostały wybrane jako zmienne decyzyjne. Do poszukiwania minimum poszczególnych kryteriów w funkcji celu wykorzystano algorytm genetyczny. W rezultacie otrzymano zestaw rozwiązań Pareto optymalnych tj. układ materiału o specyficznym współczynniku absorpcji akustycznej rozmieszczony odpowiednio na brzegach pomieszczenia.

1. INTRODUCTION

1.1 Mathematical model

In a room, after the source of sound starts to emit a signal, losses of acoustic energy caused by absorption on room's boundaries are equalized at the same time by energy from

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the source and in an enclosure acoustical steady-state is reached. In order to describe acoustic field distribution inside a room one can use a modal analysis formulation under several restrictions [1,2,3]. Using modal analysis assumptions, acoustic pressure in a room interior can be described with its normal modes (eigenfunction value) Ψ_n and closely related eigenfrequencies ω_n. Eigenfunctions for sufficiently enough high boundaries' impedance, approximately equals eigenfunctions for the same room with hard wall boundary condition. Subsequently orthogonal, normalized set of functions Ψ_n are required [1,2,3]. An acoustic field in a room with harmonic source inside describes linear, inhomogeneous wave equation with the source term $\Omega \cdot e^{j\omega t}$. The Ω describes distribution and strength of a source of a frequency ω. Under mentioned above considerations the solution of the wave equation can be represented by a sum over a set of eigenfunctions and time components[1]:

$$p(x,y,z,t) = \sqrt{V}\sum_{n=0}^{\infty} P_n(t)\Psi_n(x,y,z) \tag{1}$$

where V is a volume of a room and $P_n(t)$ are the time components i.e. the modal amplitudes. For a steady-state and harmonic source, modal amplitudes are in the form $P_n(t) = A_n e^{j\omega t}$ and its amplitudes equal:

$$A_n = \frac{Q_n}{\omega_n^2 - \omega^2 + 2j\omega r_n}\left[1 - 2j\omega\sum_{r=0} \frac{\xi_{rn} r_{rn} Q_r}{\omega_r^2 - \omega^2 + 2j\omega r_{rr}}\right] \tag{2}$$

where $\xi_{rn}=1$ for $r \neq n$ and $\xi_{rn}=0$ for $r=n$, r_n is a room's damping coefficient, r_{rn} is coupling coefficient (its value represents coupling between modes occurring with impedance condition in a room) and $Q_{n,r}$ is an acoustic source function [1], which are represented by relations:

$$r_n = 0.5 \cdot \rho_0 \cdot c^2 \int_S \frac{\Psi_n^2}{Z_S} ds \tag{3}$$

$$r_{rn} = 0.5 \cdot \rho_0 \cdot c^2 \int_S \frac{\Psi_r \Psi_n}{Z_S} ds \tag{4}$$

$$Q_n = \frac{c^2}{\sqrt{V}} \int_V \Omega \cdot \Psi_n dV \tag{5}$$

where S describes surface of a room boundaries with individual impedance Z_S. Generally, for a low frequency range, the length of an acoustic waves is much biger than source dimensions and one can consider it as a point source, a monopole excitation placed in the coordinates (x_0, y_0, z_0), mathematically written as:

$$\Omega \cdot e^{j\omega t} = q \cdot \delta(x_0, y_0, z_0) e^{j\omega t} \tag{6}$$

and the source term integral (5) is developed as:

$$Q_n = \frac{c^2}{\sqrt{V}} q \cdot \Psi(x_0, y_0, z_0) \tag{7}$$

Eventually in this approach, to obtain pressure distribution in an enclosure it is necessary to calculate values of eigenfunctions, eigenfrequencies and the integrals (3) (4).

1.2 Frequency response function (FRF)

From formulations (1) and (2) values of acoustic pressure and its distribution for certain frequency in steady state field conditions are known *p(x,y,z)*. For the unit excitation i.e. when *q*=1 in (7), reconstructed equation (1) becomes frequency response function (FRF) *p(x,y,z,ω)*. Now, it is possible to examine acoustic properties of rooms in the frequency domain.

2. OBJECTIVE FUNCTION

An acoustic pressure in each enclosure, described by (1) in the time domain or FRF in the frequency domain, are directly depended on modal amplitudes (2). Eigenfunctions Ψ_n and eigenfrequencies ω_n are constant and characteristic for a particular room. Therefore, the influence at interior acoustic field can be done by modal amplitudes modification. When constant position (x_0, y_0, z_0) of the sound source is considered, additionally factor Q_n in (2) is invariable. Eventually, damping coefficients r_n play main role in room's acoustic filed creation, per boundaries impedance values Z_S and its distribution per eigenfunctions Ψ_n. values on the specific surfaces S (3). Therefore, the minimum of an acoustic pressure in enclosure can be achieved applying maximum value (from an assumed range) of the impedance Z_S on all boundaries. On the other hand, in practice higher impedance on surface increases general costs. Thus, there are two opposite criteria and in consequence, a double criteria objective function can be considered with intention of searching optimal values of walls' impedances which give maximal reduction of acoustic pressure inside enclosure simultaneously involving minimal costs.

In order to evaluate an acoustic pressure reduction in a certain frequency range, the FRF approach was chosen. For each frequency of FRF, spatial root mean square value p_{rms} was calculated according to:

$$p_{rms} = \sqrt{\int_V \frac{p^2}{V} dv} = \sqrt{\mathrm{Re}(A_n)^2} \tag{8}$$

where *p* is a real part of pressure *p(x,y,z,ω)*. Sequentially, mean and standard deviation values of the function $p_{rms}(\omega)$ were calculated. For that reason, the first criterion K1 (acoustic criterion) states:

$$K1 = \bar{p}_{rms}(\omega) \cdot \sigma_{p_{rms}(\omega)} \rightarrow \min \tag{9}$$

The second criterion K2 (cost criterion) states: values of impedances of particular surfaces (walls, a ceiling, a floor) have to be close to the highest impedances from the examined range. Additionally, each separate surface, where the impedance could vary, was related to its weight w_i. Values of the weights reflect the relative importance of the surface in enclosure and are related to its area. Finally, the cost criterion is of the form:

$$K2 = \sum_{i=1}^{m} w_i (Z_{\max} - Z_i) \rightarrow \min \tag{10}$$

where *m* is a number of surfaces taken into considerations. This is a representation of a linear function where costs increase with increasing a distribution area and decreasing an impedance of the material.

3. OPTIMIZATION AND GENETIC ALGORITHM METHOD

A general multi-objective procedure can be apply for this problem as follows:

$$\begin{aligned} &\underset{Z_i}{Minimize}\, F(Z_i) = [K1(Z_i), K2(Z_i)]^T \\ &subject\ to\ Z_i - \mathrm{Z}_{\max} \le 0 \wedge \mathrm{Z}_{\min} - Z_i \le 0,\ \mathrm{i} = 1,2,...\mathrm{m} \end{aligned} \tag{11}$$

This is a double-objective function with *2m* inequality constraints and *m* design variables (decision variables). Set of all admissible values Z_i (values of an impedance) states the feasible design space Z. Set of the objective function values $F(Z_i)$ for all the design variable values from the design space Z is the feasible criterion space F.

In a case of the multi-objective optimization, like in our problem, there is no single global solution. It is necessary to determine a set of *n* solutions $\{[Z_1,Z_2,\dots,Z_m]_n\}$from Z, that fit a predominant definition of an optimum, and then to compose Pareto optimal solutions [5]:

Solution $Z^O=[Z_1^O,Z_2^O,\dots,Z_m^O]$ from Z is Pareto optimal iff there does not exist another solution $Z^*=[Z_1^*,Z_2^*,\dots,Z_m^*]$ from Z, such that $F(Z^*)\le F(Z^O)$, and $K(Z^*)<K(Z^O)$ for at least one of the criteria.

In order to examine the objective function, taking into account two criteria, the genetic algorithm method [4,5] was used. Simultaneously, such a method is considered as one of the direct stochastic multi-objective optimization method. It works under a selected population (*generation*) of a feasible design space Z (sets of design variables (*genes*), particular number of *individuals*, *chromosomes*) at the same time, relating to solutions in a feasible criterion space F. From that solutions are taken out the best-Pareto-optimal. The described procedure is iterative one.

Therefore, the first problem is how to incorporate Pareto optimality evaluation of a potential solutions calculated for a particular, following populations (*generation*) and select proper individuals (*selection*). The rank algorithm which assigns number (rank) to each individual in a population is used here. All non-dominated individuals (fit a predominant definition) receive the rank=1. Then this individuals are temporary remove from population and evaluation process starts again. This time non-dominated individuals obtain the rank=2. The procedure repeats until all individuals get a rank. Next ranks are recalculated and individuals with the rank=1 get maximal according with formula: max(rank)+1-rank. Thus, individuals with the rank=1 get max(rank), with the rank=2, max(rank)-1 and "the worst" individuals get 1. Eventually, the roulette wheel method is used. This is a linear representation of this method which applies a random number generator. It returns pseudo-random values drawn from a uniform distribution on the unit interval and then it multiplies by the sum values of recalculated ranks. Subsequently, the random number is compared with a sum of successive recalculated ranks starting from lowest value i.e. 1. When the sum exceeds the random value, index of the last added, points out a chromosome (individual) of parent in the next generation (population). The roulette procedure is applied until the counter reaches a number which is equal half of the population size. Finally, the number of Pareto-optimal individuals is limited by calculating distances between each other. Individuals with the smallest distances are removed. In this way one can obtain a proper distribution and spread of Pareto-set.

The second problem is how to provide a potential possibility that all individuals will be taken into consideration, on the one hand (*crossover* and *mutation*) and the best individuals will continue or transfer its feature (*genes*) to the next population (*elitism*). Here, blending crossover is applied, which is represented by formula:

$$Z^{C1} = \varepsilon \cdot Z^{P1} + (1-\varepsilon) \cdot Z^{P2}$$
$$Z^{C2} = \varepsilon \cdot Z^{P2} + (1-\varepsilon) \cdot Z^{P1} \tag{12}$$

were ε is a random number between 0 and 1, Z^{C1}, Z^{C2} chromosomes of the children (create next generation) and Z^{P1},Z^{P2} chromosomes of the parents (previous generation). This means that the chromosomes' genes of the children $Z^{C1}=[Z_1^{C1},Z_2^{C1},\ldots,Z_m^{C1}]$, $Z^{C2}=[Z_1^{C2},Z_2^{C2},\ldots,Z_m^{C2}]$ take values between the values of the corresponding genes of the parents $Z^{P1}=[Z_1^{P1},Z_2^{P1},\ldots,Z_m^{P1}]$, $Z^{P2}=[Z_1^{P2},Z_2^{P2},\ldots,Z_m^{P2}]$.

$$\begin{gathered} Z_1^{P1} < Z_1^{C1} < Z_1^{P2} \\ Z_2^{P1} < Z_2^{C2} < Z_2^{P2} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ Z_m^{P1} < Z_m^{C2} < Z_m^{P2} \end{gathered} \tag{13}$$

The mutation guarantees that stochastically each chromosome can be evaluated and it is possible to reach a minimum of the objective function. It means random change of a gene value of a randomly chosen chromosome. Here 30% of the chromosomes are mutated.

Such an approach was applied due to the following advantages: a genetic algorithm do not require a gradient information which could be difficult to get in a case of large number of a design variables, a nature of the objective function is not known, a genetic algorithm converges to the global solution rather than to a local one, Pareto-optimal solution are available directly, an initial population which is generated using uniform distribution guarantee covering the whole feasible design space with equal probability.

3. EXAMPLE

3.1 Sample object

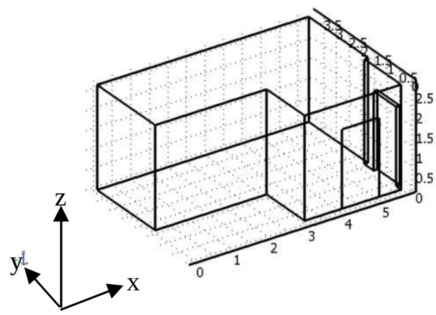

Fig.1. Shape and dimensions of the examined object

As an example of optimized object, the room shown in Fig.1, is taken into consideration. The volume of the enclosure is 45,27 m^3, and total surface area S with varying impedance is 84,96 m^2. 15 different surfaces are considered (walls, the floor, the ceiling, doors). Subsequently, the double-objective function has been created using the relations (9) and (10). The minimization procedure (11) has been applied where the impedance Z_i of 15 surfaces (m=15) varies from $Z_{min}=5\cdot 10^4$ to $Z_{max}=10^6$ Pa•s/m. The modal amplitudes A_n have been found according to relation (2), where the first 500 (n=500) modes are involved which is related to the eignfrequency 480,4Hz as a limit. Eigenfunctions Ψ_n, eigenfreqenties ω_n and factors Q_n, r_{nm}, r_n have been obtained numerically using FEM method. Furthermore, spatial root mean square acoustic pressure values p_{rms} for frequency response function FRF have been evaluated. Eventually, the aim of the acoustic criterion K1 (9) was to obtain the most flat FRF with lowest mean in the low range of frequency (30Hz-500Hz). In the case of the criterion K2 (10) the weights w_i are defined to emphasize the surfaces of small area and its sum equals unity ($\Sigma w_i = 1$).

3.2. Results

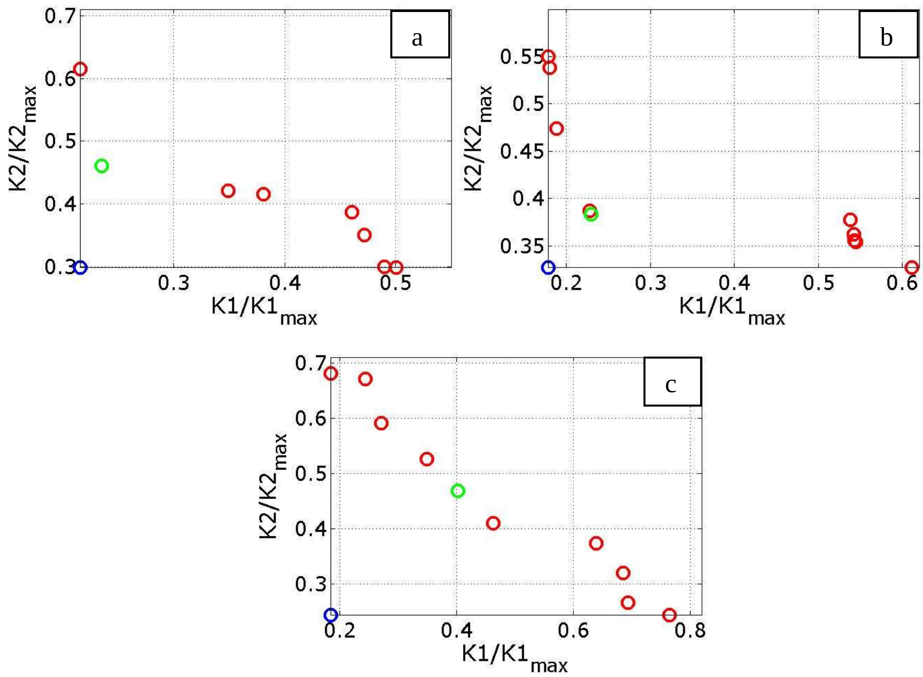

Fig.2. Pareto-optimal solutions. Criterion values related to values for maximal values for source position: a)-x_0=1,08m, y_0=2,51m, z_0=1,43m; b)-x_0=3,0m, y_0=2,52m, z_0=1,33m; c)- x_0=4,5m, y_0=2,51m, z_0=1,31m;

Results were obtained for three different sound source positions (x_0,y_0,z_0). The genetic algorithm was adjusted for the following options: the population size-10, the number of iterations-25 and design variables tolerances 10^2 Pa·s/m. The tolerance means that the design variables were varied with exact precision.

In Fig.2 Pareto-optimal solutions for values of criteria related to corresponding maximal values i.e. $K1_{max}$, calculated for individual $Z=[Z_1=Z_{max}, Z_2=Z_{max},…,Z_{15}=Z_{max}]$ and $K2_{max}$ for $Z=[Z_1=Z_{min}, Z_2=Z_{min},…,Z_{15}=Z_{min}]$ are shown. Thus, one can directly read, how high is the spatial root mean square acoustic pressure reduction due to the applied acoustic absorption material and which cost is exact part of maximal feasible cost. Subsequently, one can make a decision which solution should be taken into consideration. Depending on which criteria is the most significant, solutions with particular values of the criteria will be chosen. If low costs are preferable, solutions with low values K2 should be considered. If high pressure reduction is to achieve, the solution with low K1 values will be taken.

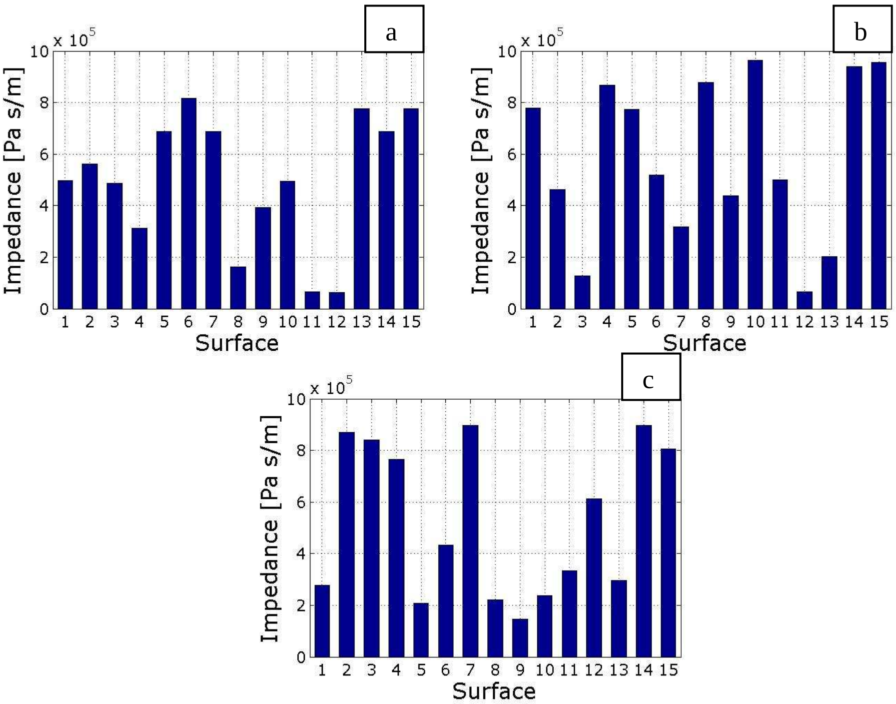

Fig.3. Values of design variables (surface impedance) for Pareto-optimal solutions closest to utopia point, sorted in increasing order of areas, for source position: a)-x_0=1,08m, y_0=2,51m, z_0=1,43m; b-x_0=3,0m ,y_0=2,52m ,z_0=1,33m; c)-x_0=4,5m, y_0=2,51m, z_0=1,31m;

In Fig 3, the one particular solutions, selected from the Pareto-optimal is shown. It is specific point (o-green points in the Fig.2), selected by the evaluation of its distance from utopia point (o-blue points in the Fig.2) [5]. This point is unattainable and lies out of the feasible criterion space F. In this case it is a semi-utopia point of coordinates ($K1_{min}$, $K2_{min}$). In Fig.3 room's surfaces are put in increasing area order. According to the cost criteria K2, impedances of the biggest surfaces should take values in a high range of the impedance. This is clearly seen in Fig.3.

4. CONCLUSIONS

The procedure presented in the paper can be used for other applications in the room acoustics and another systems, especially in the cases where many factors should be taken into consideration. Genetic algorithms appears also as a proper tool in at least two areas of optimization problems. First is a fast means finding minimum of single objective functions, which character is not exactly known, with many arguments in addition. Secondly, it is direct method of multi-objective optimization as well. It works effectively especially in a case of conflicted criteria problems. As the result of genetic algorithm procedure, the set of optimal solutions available directly (Fig. 2) and one can decide which solution is the suitable one (Fig. 3). Based on the results obtained for the sample object one can make following suggestions:

- Each point in Fig.2 represents set of 15 values of the design variables space Z, i.e. an impedance on a particular surface (Fig.3). In order to find Pareto-optimal solutions, using full survey method one needs huge computational resources and time with comparison to genetic algorithms;
- The range of Pareto-optimal solutions depends on the source position. For a particular source position in the room there are limits of an optimal application of an acoustic absorbing material, bounded by the value of costs (Fig.2). It means that an increase of the impedance of absorbing material and an area of its distribution or decrease values of those factors, exceeding optimal costs bounds as the result, one will not achieve a significant acoustic pressure reduction on the one hand or values of the spatial root mean square of an acoustic pressure will increase excessively on the other hand. Similar evaluation is also valid when one take second criteria into consideration. The optimal bounded range of pressure reduction exist as well. Thus, the whole range in which one can optimally influence on the system is known;
- The Pareto-optimal curve (Fig.2) takes different shapes for different source position.It means that acoustic properties are varied with this factor. From the shape of Pareto curve one can deduce, in which of its range one can influence the system effectively;
- One should be aware of stochastic character of genetic algorithms. Its capability to find an optimal solution thanks to internal algorithms like a selection, a crossover and a mutation make possible save the time by working on a bounded population but it needs good knowledge how to choose a size of the population and iteration.

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