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MARKOV MODEL OF THE SHIP'S NAVIGATIONAL SAFETY ON THE OPEN WATER AREA

Abstract: In the paper the navigational safety model for ship on the open area Has been proposed. The Markov process has been used to describe the safety model. Futhermore, the characteristics of this model have been determined.
Keywords: navigational safety, open area, Markov process

1. INTRODUCTION

The navigational safety is based primarily on analysis of current situation and depends on the interaction between the man, technology and environment [1], [2] . The literature mentions that the errors at any of components have the greatest impact on the occurrence of hazardous situations [1], [3]. Therefore, the methods and criterions to settle the hazard situation and to evaluate of quality control and assessment in terms of traffic safety ([3], [1], [2], [4], [5]) is very important. It could help to develop the best control or the best manoeuvres for given hazard situation ([1], [3], [5], [6],[7], [8]). Thus, the ability to anticipate the navigation of a hazardous situation [4] is possible.
For better analysis of the hazard situation at sea, two basic measures - the CPA and TCPA ([9]), for estimation of closer distance between ships were introduced. Another important approach to the risk of the ship is domain analysis ([1], [6]).
The article attempts to describe the stochastic model as a tool to analysis of the hazard situation. The Markov processes ([5], [10]) are used to describe the main characteristics of the considered model.

2. THE NAVIGATIONAL SITUATION RANDOM DYNAMIC MODELLING

The main part of the analysis of particular navigational situation is an observation. During this process the navigator's focus on direction where the potential danger is the highest. Therefore the area of observation can take any symmetric or asymmetric shape and take different values of the collision probabilities. Hence, the concept of the random map of hazards was introduced in [11].

2.1. BASIC NOTATIONS

The model of the ship on the waterway is the operation process. In article, we used the following notations [11], [12]:

X_{ij} – is two dimensional binary random variable representing random state of component e_{ij}, which is equal to 1, when component e_{ij} is free and is equal to 0, in the other case, $i = 1,2,...,n,\ \ j = 1,2,...,l_i$,

$p_{ij} = P(X_{ij} = 1)$ – is the probability of event that component e_{ij} is free, $i = 1,2,...,n$, $j = 1,2,...,l_i$,

$q_{ij} = 1 - p_{ij} = P(X_{ij} = 0)$ – is the probability that component e_{ij} is busy, for $i = 1,2,...,n,\ \ j = 1,2,...,l_i$,

$A^l_{k,n}$ – is the event, when the k consecutive blocks out of n are free,

$P^l_{k,n} = P(X^l_{k,n} = 1)$ – is the probability of event $A^l_{k,n}$,

$\overline{P}^l_{k,n} = 1 - P^l_{k,n} = P(X^l_{k,n} = 0)$ – is the probability that the event $A^l_{k,n}$ is not true.

r_{ij} – is importance coefficient of component e_{ij}, for $i = 1,2,...,n,\ \ j = 1,2,...,l_i$.

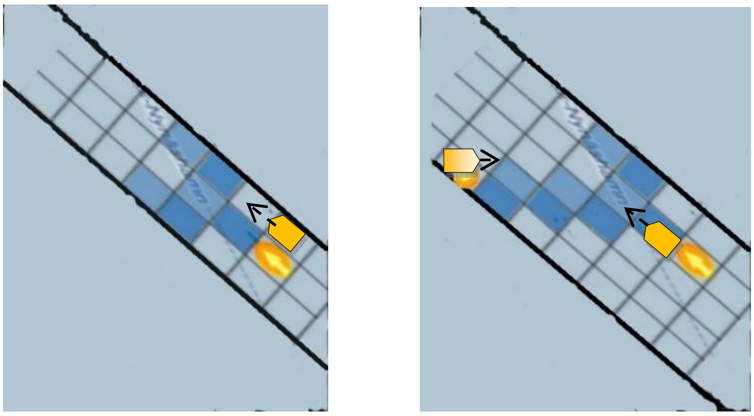
2.2. OPERATION MODEL

It is required to give the following definitions [11], [12].

Definition 1. The operational block Y_j, j=1,.., n is the set of cells e_{ij}, for $i = 1,2,...,n$, $j = 1,2,...,l_i$, of waterways designating specified geometric shape.

Then the system is defined as follows.

Definition 2. The system is the ship and the set of *n* equal consecutive operational blocks in front of ship.



A B

Figure 1. Operational models A) symmetrical case; B) asymmetrical case

Definition 3. The number of *k* consecutive free operational blocks, for the current location of ship, is called the system state.

The operation model is consist of the system defined at Definition 2, where shape of the area is determined by navigational situation.

There is defined the three state navigational safety model, with safety state, safety threat state and danger state [11], [12].
The safety is a state in which the movement of the vessel shall be continued without the need for unscheduled change of course.
The safety threat is a state in which there is a need to perform an unplanned manoeuvre change course.
The danger is a state in which it is necessary changes in the course of rapid manoeuvre.
These states are determined by the following subsets of operational states:
$S_B = \{s_i; i \geq n_{ZB}\}$ for the safety state;
$S_{ZB} = \{s_i; n_{NB} \leq i < n_{ZB}\}$ for the risk state;
$S_{NB} = \{s_i; i < n_{NB}\}$ for the danger state, where parameters n_{ZB} and n_{NB} are the limits values for numbers of free operational blocks.

We get the following theorem [11].
Theorem 1. The probability, that at least k consecutive blocks of waterway out of *n* is free, is given by the following formula

$$P(A^{l}_{k,n}) = \left[1 - \prod_{j=1}^{l_{k+1}} p_{k+1\,j}\right] \prod_{i=1}^{k} \prod_{j=1}^{l_i} p_{ij}, \quad k \in N. \tag{1}$$

3. MARKOV MODEL OF THE SHIP ON THE WATERWAY

Suppose $P(S_B)$, $P(S_{ZB})$, $P(S_{NB})$ means the probability of being in the respective states of the system safety model. Then the probability, that the vessel is in the safe state, equals to the probability that at least n_B consecutive sections of the waterway out of n before it is free, i.e.

$$P(S_B) = P(A^l_{n_B,n}). \tag{2}$$

Next, the probability, that the vessel is in the risk state, equals to the probability that at least n_{ZB} and at most n_B consecutive sections of the waterway out of n before it is free. So taking into account previous models we get:

$$P(S_{ZB}) = P(A^l_{n_{NB},n}) - P(A^l_{n_{ZB},n}). \tag{3}$$

The probability, that the vessel is in the danger state, equals to the probability that at most n_{ZB} consecutive sections of the waterway out of n before it is free. So taking into account previous equations we get

$$P(S_{NB}) = 1 - P(A^l_{n_{NB},n}). \tag{4}$$

A random process in which the probabilities of states in a series depend only on the properties of the immediately preceding state or the next proceeding state, independent of the path by which the preceding state was reached. It is distinguished from a Markov chain in that the states of a Markov process may be continuous as well as discrete.
For the safety model presented in the earlier section, the schema of transitions is given in Figure 2.

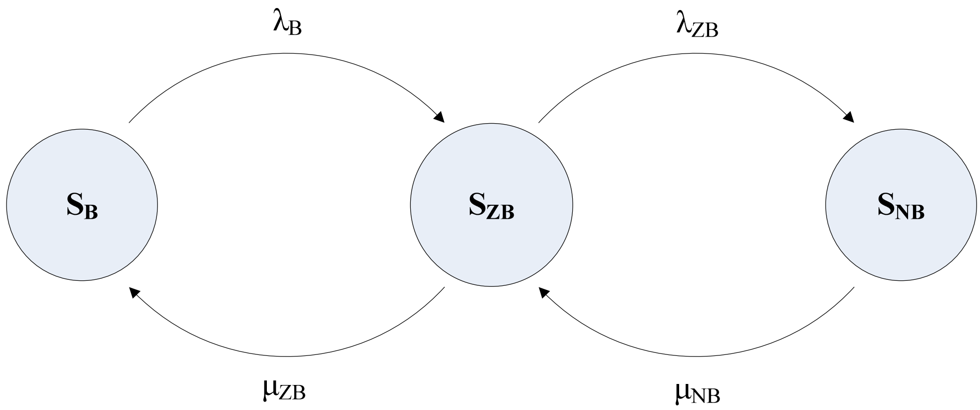


Figure 2. The schema of the Markov safety model of the ship on the waterway

Intensity of transition between the states (μ_{ZB}, μ_{NB}, λ_B, λ_{ZB}) are calculated using the operation model from (2) – (4).
The Markov process of changing the safety states, what is continuous in time, can be described by the Kolmogorow's equations, in form [5]:

$$\begin{cases} P_B^{/}(t) = -\lambda_B P_B(t) + \mu_{ZB} P_{ZB}(t) \\ P_{ZB}^{/}(t) = \lambda_B P_B(t) + \mu_{NB} P_{NB}(t) - (\mu_{ZB} + \lambda_{ZB}) P_{ZB}(t) \\ P_{NB}^{/}(t) = \lambda_{ZB} P_{ZB}(t) - \mu_{NB} P_{NB}(t) \end{cases} \tag{5}$$

We assume, that the system (5) is in safety state at the moment *t=0*, i.e. the initial distribution is given by the vector [1,0,0], and applied the Laplace's transform, we get the following solutions:

$$\begin{aligned} P_B(t) = \left(\frac{\mu_{NB}\mu_{ZB}}{s_1 s_2}\right)\chi(t) + \left(\frac{-1}{s_2 - s_1}(s_1 + \mu_{ZB} + \lambda_{Zb} + \mu_{NB}) - \frac{\mu_{NB}\mu_{ZB}}{s_1 s_2}\right)\exp\{s_1 t\} \\ + \left(\frac{1}{s_2 - s_1}(s_2 + \mu_{NB} + \lambda_{ZB} + \mu_{ZB}) + \frac{\mu_{NB}\mu_{ZB}}{s_2}\right)\exp\{s_2 t\} \end{aligned} \tag{6}$$

$$\begin{aligned} P_{ZB}(t) = \left(\frac{\mu_{NB}\lambda_B}{s_1 s_2}\right)\chi(t) + \left(\frac{-\lambda_b}{s_2 - s_1}\left(1 + \frac{\mu_{NB}}{s_2}\right) - \frac{\mu_{NB}\lambda_B}{s_1 s_2}\right)\exp\{s_1 t\} \\ + \left(\frac{-\lambda_b}{s_2 - s_1}\left(1 + \frac{\mu_{NB}}{s_2}\right)\right)\exp\{s_2 t\} \end{aligned} \tag{7}$$

$$P_{NB}(t) = \left(\frac{\lambda_{ZB}\lambda_B}{s_1 s_2}\right)\left[\chi(t) + \left(\frac{-s_1}{s_2 - s_1} - 1\right)\exp\{s_1 t\} + \left(\frac{-s_1}{s_2 - s_1}\right)\exp\{s_2 t\}\right], \tag{8}$$

Where $\chi(t)$ is the Heaviside step pdf and the s_1 i s_2 are the roots of the polynomial

$$s^2 + (\lambda_B + \mu_{NB} + \lambda_{ZB} + \mu_{ZB})s + \lambda_B \mu_{NB} + \lambda_B \lambda_{ZB} + \mu_{ZB}\mu_{NB} = 0 \tag{9}$$

given in the following form

$$s_1 = \frac{1}{2}\left[-(\lambda_B + \mu_{ZB} + \lambda_{ZB} + \mu_{NB}) - \sqrt{1 + \frac{4\lambda_{ZB}\mu_{ZB}}{(-\lambda_B - \mu_{ZB} + \lambda_{ZB} + \mu_{NB})^2}}(-\lambda_B - \mu_{ZB} + \lambda_{ZB} + \mu_{NB})\right] \tag{10}$$

and

$$s_1 = \frac{1}{2}\left[-(\lambda_B + \mu_{ZB} + \lambda_{ZB} + \mu_{NB}) - \sqrt{1 + \frac{4\lambda_{ZB}\mu_{ZB}}{(-\lambda_B - \mu_{ZB} + \lambda_{ZB} + \mu_{NB})^2}}(-\lambda_B - \mu_{ZB} + \lambda_{ZB} + \mu_{NB})\right]. \quad (11)$$

4. CONCLUSIONS

The proposed operational and safety model have taken into account the navigator's subjective minimum level of safety acceptance.
These models can be more complicated but more flexible and better suited to real situation, when the semi-Markov model of the operational process will be given instead of the Markov model.

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MARKOWSKI MODEL BEZPIECZEŃSTWA NAWIGACYJNEGO STATKU NA AKWENIE OTWARTYM

Streszczenie: W artykule przedstawiono model bezpieczeństwa nawigacyjnego dla statku na akwenie otwartym. Do opisu tego modelu użyty został proces markowski.. Dodatkowo dla tego modelu obliczono jego charakterystyki.

Słowa kluczowe: bezpieczeństwo nawigacyjne, akwen otwarty, proces markowski