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# RAIL IRREGULARITIES IDENTIFICATION WITH USE OF INVERTED PARAMETRIC MODELS

**Abstract**: The paper presents an application of the loading force identification method based on inversion of regressive parametric models to the reconstruction of rail irregularities. The irregularities are identified from the accelerations of vibrations signals measured on the axel boxes of the car during its ride. In the article both numerical and experimental verification of the method are shown.
**Keywords**: rail irregularities identification, inverse problem, parametric models

## 1. RAIL IRREGULARITIES

Rail irregularities are generated by series of factors caused by both operation and ordinary rail degradation. Initiation and growth of rail irregularities has many negative consequences. First of all, high level of rail irregularities threatens the safety of the rail traffic, throughout increase of a derailment probability, increase of impact forces level and decrease of durability of all the components of the track-vehicle system. Second of all it decreases ride comfort and increases a negative influence on the environment. It happens because rail irregularities generate vibrations, perceived by passengers of the rail vehicles, and noise emitted to the environment. Due to the reasons there is a necessity of monitoring of the rail irregularities development. Such a monitoring is presently carried out, but applied techniques are very costly, and it further results in low frequency of performed measurements. Another factor that limits the frequency of rail irregularities measurements, is necessity of traffic exclusion on the tested route, what, for the tracks with heavy traffic, could be difficult.

In view of the importance of the issue and the difficulty of its solution, authors propose to identify the rail irregularities as a kinematic excitation on the basis of the system response measured on the unsprung elements of the rail vehicles (e.g. axle boxes). Such a response in form of vibrations accelerations are easy for measurement, do not require a special vehicles and the measuring equipment is not very expensive. One can imagine that the measuring system for the vibrations accelerations measurements could be installed in selected trains, and data from that system would be used for rail irregularities identification. As it was stated such a solution is much chipper and less difficult from the organization perspective (no necessity of traffic exclusion on the tested route). Because of these advantages rail monitoring could be performed much more often.

## 2. PROBLEM FORMULATION

Rail irregularities identification based on the vibrations accelerations measured on the vehicle is an example of inverse problem defined in the following way: model of the system is known as well as the response of the system. Kinematic excitation in form of the rail irregularities is to be identified. As it was mentioned above it is a very complex problem due to the fact that it is non-linear and non-collocated. The problem has to be solved in the time domain. To its solution the method of quality function minimization was proposed [11]. This method however is very time consuming and for that reason rather not practical. Now the authors propose another method of excitation reconstruction that can operate even in real time.

Reconstruction of the input of the system by inverting the system's model is important in multiple applications. Input reconstruction is a technique frequently used in the Internal Model Control (IMC) strategies [6] to invert data-driven parametric models and compensate the dynamics of the tracking process, or for metrological purposes [2]. The literature, however, rarely addresses the problem of dynamic inversion [3] based on data-driven parametric models of mechanical structures and systems. Nonetheless, the technique (model inversion) is applicable to the problems of load reconstruction in mechanical systems in order to modify the dynamics of a structure or a system and to achieve better performance, e.g. to lower the level of loading forces [10]. Load prediction in systems for which the force signal cannot be directly measured due to constructional constraints, as in the case of forces being exchanged by a wheel and the road or rail, is considered to be one of the most practical applications of the inverse approach [1].

In a parametric approach, a model has the form of a transfer function or a system of state-space equations. Recollecting from the preceding section, a linear and time invariant system with a single input and a single output (SISO) can be represented as an input-to-output transfer function. The role of the inverse model is to filter the response of the system in order to reconstruct the unknown input. It can be shown, cf. [5, 7], that the transfer function inverse to G is the reciprocal of G, i.e. the ratio

$$G^*(z) = \frac{1}{G(z)} = \frac{u(z)}{y^*(z)} = \frac{a_0 + a_1 z + \ldots a_{nA} z^{nA}}{b_0 + b_1 z + \ldots b_{nB} z^{nB}} \tag{1}$$

It can be seen that the inversion maps zeros of the G(z) onto poles of the $G^*(z)$ and, accordingly, poles of the G(z) onto zeros of the $G^*(z)$. The transfer function is strictly proper if the degree of the numerator is less than the degree of the denominator $nB<nA$, while it is proper if the degree of the numerator is equal to the degree of the denominator, i.e. $nB=nA$. If the degree of the numerator is greater than the degree of the denominator,

i.e. $nB>nA$, the transfer function is called improper [5, 7]. The inversion involves the equality of the reference input $u^*(z)$ and input $u(z)$ by introducing a reference transfer function $G_r(z)$

$$G_r(z) = \left( \frac{1-T_d}{z-T_d} \right)^{(nA-nB)} \tag{2}$$

Assuming that $G_r(z)$ compensates $nA$-$nB$ poles of the denominator of the transfer function G(z), the inversion implies equality of the reference input $u(z)$ to the reconstructed input $y^*(z)$ as follows. The transfer function $G^*(z)$ inverse to the G(z) is given by [5, 7]

$$G^*(z) = \frac{G_w(z)}{G(z)} = \frac{u(z)}{y^*(z)} = \frac{a_0 + a_1 z + \ldots a_{nA} z^{nA}}{b_0 + b_1 z + \ldots b_{nB} z^{nB}} \left( \frac{1-T_d}{z-T_d} \right)^{(nA-nB)} \tag{3}$$

where: $G^*(z)$ is the inverse transfer function, and G($z$) is the direct transfer function; $u(z)$ is the reference input, $y^*(z)$ is the output, $T_d$ is the constant defining the quality of the inversion for signals of high-frequency content, and $nA$ and $nB$ are the degrees of the polynomials [7]. The inverse transfer function is unstable if at least one of the zeros of the direct transfer function is located outside the unit circle or inside the unit circle, for $z^{-1}$ or z operators respectively. These zeros create a non-minimum phase transfer function, and hereafter are referred to as non-minimum phase zeros. The occurrence of non-minimum phase zeros is caused by one the following factors: (i) the sampling interval is too short, (ii) the discrete time delay is too long, or (iii) the number of poles, in comparison to the number of zeros, is too high. It is clear from equation (3) that the inverse transfer function is unstable because non-minimum phase zeros become unstable poles. The inverse transfer function can be stabilized, however, by factorization of the numerator B(z), as discussed in [9].The advantage of such a stabilization method is the lack of phase error and delay, while the only disadvantage is a small gain error that is, moreover, negligible if the output signal consists of low frequency components [9]. This stabilization technique was applied to load reconstruction by [1].

## 3. SIMULATION VERIFICATION

The model of the system was built in Adams multi-body software package. Below the main details considering the model used for simulation are gathered together. The model was composed of 7 bodies: **c**ar body, axels with wheels (wheel sets) (2), axle boxes (4).
The following connections were modeled: axel boxes with axels connected with use of the revolute joints and car body with axel boxes connected with use of vertical springs and dampers (one spring and damper per one axel box).
The model parameters, that are masses of the bodies and stiffness coefficients, were chosen to meet the data of the existing car of Fals series, type 665 4 011-4. This model of the car was selected because it was also used for the data collection for experimental verification of the method presented in the next section of the paper.

Simulations were performed in Adams Vi Rail software package, which is dedicated for rail oriented applications. During following simulations there were different rail irregularities profiles applied and car was run with different velocities. In each simulation virtual sensors of measured vibrations accelerations were available on the car axle boxes. These data were next used for irregularities identification. Simulation verification was then performed according to the following procedure: (i) rail irregularities profile definition, (ii) simulation of the run of the car on the rail with irregularities defined in the first step, (iii) partition of the data sets into data used for parametric model identification of the process and data used for validation, (iv) parametric model identification and its inversion, (v) rail irregularities identification, (vi) comparison of the defined irregularities with identified ones.
A block diagram of the procedure of inverting a model is presented in Fig 1.

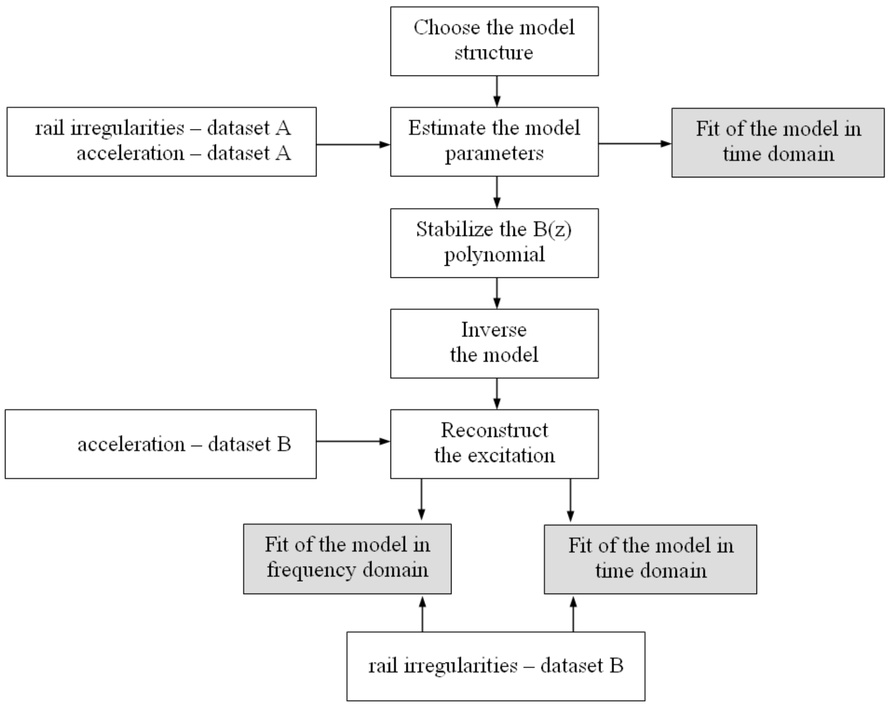


Fig. 1. Block diagram representation of the model inversion procedure based on the simulated data

The process of selecting an adequate model structure and an algorithm for estimating model parameters, the initial step, is discussed in previous sections. The next step, estimation of the parameters of the selected model, that is, estimation of the coefficients of the polynomials $A(z^{-1})$, $B(z^{-1})$, etc. from simulated data, was obtained from the Matlab System Identification Toolbox. Stabilization of the polynomial $B(z^{-1})$ is an important stage of the inversion procedure and is achieved by reflecting the roots of $B(z^{-1})$ that have a magnitude greater than unity with respect to the boundary of the unit circle (i.e. to the inside of the unit circle). Upon stabilization, the model becomes invertible directly if the degree of the numerator of the input-to-output transfer function $G(z^{-1})$ is equal to the degree of its denominator, i.e. if the model is proper. If this is not the case, a reference transfer function (Eq. 2) has to be used to compensate for the lower order of the numerator.

The performance of the inversion algorithm can be evaluated either by visual inspection of the plot of the u(i) and $u^*(i)$ or by analyzing the value of the value of the Pearson's product-moment correlation coefficient.

Inversion of the model was conducted according to the scenario presented in section 2: simulated irregularity of the railway track is required at the input of a direct model and simulated acceleration is required at its output. The orders of particular polynomials were selected using the quality indicator of the direct model's fit to the data in the time domain (Table 1).

Table 1

**Criteria of the model structure optimality**

| Criterion | Measure | Values |
|---|---|---|
| Fit in the frequency domain | Fit measure | Reconstructed input (output of the inverse model) |
| Fit in the time domain | Fit measure | Output of the direct model |
| Statistical properties of model residuals | FPE and AIC | Residuals of the direct model |
| Representation of modal properties | Subjective evaluation | Bode diagram and spectrum of residuals; both obtained from the direct model |

The adequacy of model structures was then evaluated by means of two measures, referred to in the literature as the Final Prediction Error (FPE) and the Akaike Information Criterion (AIC). Additionally, in order to detect the presence of abnormalities in modal properties in the frequency domain, a visual inspection of the Bode plot of the input-to-output transfer path and the spectra of model residuals (the disturbance-to-output transfer path) was performed for each identified model structure. Analysis of the extensive quantity of such visual indicators (not presented here due to a lack of space) indicates the presence of no abnormalities. Visual evaluation of model quality might also be supported by pole-stability diagrams plotted as a function of the orders of selected polynomials. The major criterion for polynomial orders selection was, however, the comparison of the fit quality of the reconstructed inputs. The purpose of selection is to obtain a suitable inverse filter capable of providing the best possible reconstruction of the input signals with respect to the optimality criteria listed in Table 1. The strategy implemented for optimizing selection of the model structure was the systematic search for a set of model structures that would satisfy the criteria listed in Table 1, as well as selection of constraints resulting from a priori information on delays in each particular measured response, e.g. a place where excitation is applied and a place where it is measured. To compare the results of direct measurements and those obtained with the procedure of model inversing, the correlation coefficient were computed for both the lateral and vertical forces. The results of this comparison are presented in Table 2.

Table 2

**Comparison of the measured and the reconstructed irregularities of the rail**

| Force | Correlation coefficient | |
|---|---|---|
| | right side of the car | left side of the car |
| Lateral force | 85% | 87% |
| Vertical force | 89% | 93% |

Beside Table 2, the exemplary results of load reconstruction are presented graphically in Fig. 4 in the frequency domain in linear and decibel scale respectively. All signals were scaled to zero mean and unity variance before the signal processing.

## 4. EXPERIMENTAL VERIFICATION

Because authors did not have possibility to use the equipment for rail irregularities measurements, they decided to verify the method by identification of excitation in form of rail – wheel contact forces. Because kinematic excitation in form of rail irregularities is directly interconnected with rail - wheel contact force, authors assumed that performed change in verification process is acceptable. The measurements were taken on the self-dumping car of the Fals series, type 665 4 011-4. During tests the car was empty. In Fig. 2, the placement of accelerometers on the axle-boxes is showed. During test rides time histories of two forces were recorded: vertical and horizontal both acting in the rail – wheel contact point in the first wheelset at the right side. Together with the forces, 6 vibrations accelerations measured on the axle boxes and the frame of the car where stored. Additionally information about car velocity and its gyroscopic moments was collected for the ride profile recognition.

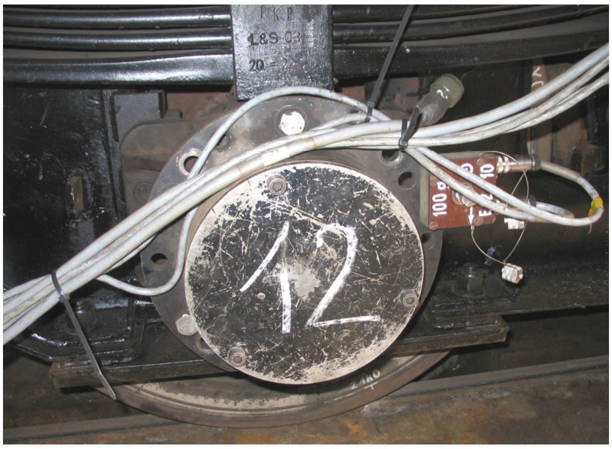

Fig. 2. Measurements of the vibrations accelerations on the axle box of the car

Forces were measured in indirect form during the tests. Displacements of the axles of the car were the measured quantity. They were measured with use of the strain gauges set and further recalculated to obtain the bending moments of the axles. These moments were next transformed into the rail – wheel contact forces. Values obtained in the presented method

were compared with the identified ones. Eight data sets were collected which corresponded to the eight rides of the car with different velocities, on the rail with varying quality. Each test ride less 20 s. Sampling frequency was set to 150 Hz.
A block diagram of the procedure of inverting a model is similar to that one presented in Fig 1. The rail-wheel force contact is the direct model input instead of the displacement representing irregularities of the railway track. The model output is the acceleration measured at the journal box.
Inversion of the model was conducted according to the scenario where the measured random force is required at the input of a direct model and measured acceleration is required at its output. The orders of particular polynomials were selected using the quality indicator of the direct model's fit to the data in the time domain in the same manner as presented for the simulation case. To compare the results of direct measurements and those obtained with the procedure of model inversing, the correlation coefficient were computed for both the lateral and vertical forces. The results of this comparison are presented in Table 3.

Table 3

**Comparison of the measured and reconstructed rail-wheel contact force**

| Force | Correlation coefficient | |
|---|---|---|
| | right side of the car | left side of the car |
| Lateral force | 65% | 63% |
| Vertical force | 71% | 72% |

Beside Table 3 the exemplary results of load reconstruction are presented graphically in Fig. 4 in the frequency domain in linear and decibel scale respectively. All signals were scaled to zero mean and unity variance before the signal processing.

## 5. RESULTS AND CONCLUSIONS

The paper addresses questions concerning the feasibility of reconstructing the excitation of the mechanical systems exposed to kinematic excitation which is difficult for direct measurement. The purpose of this work is to advocate model inversion based on a parametric system identification approach, as an alternative method for approaching this class of problems to a non-parametric one presented by Uhl [10].

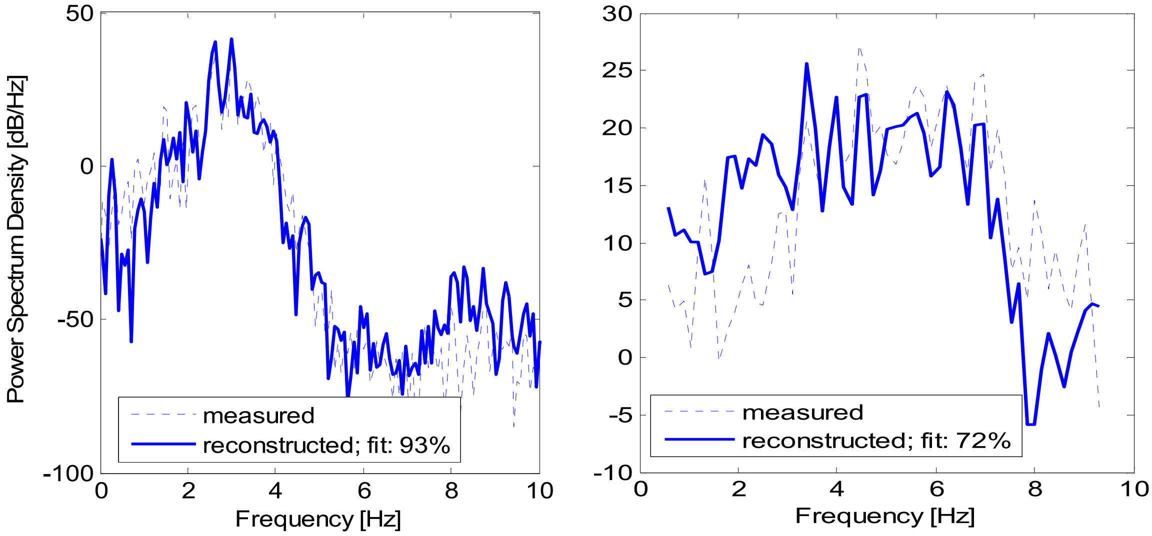


Fig. 3. Results of load reconstruction in the frequency domain model (i) left BJ(25,1,17,25) (simulations) and (ii) right ARX(33,29) (measurements)

The paper presents the theory and discusses case studies of inverting data-driven models of mechanical system. Numerical and experimental validation tests confirm that the methodology proposed herein, i.e. parametric system identification and model inversion, is valid for both simulation and operational data (Fig. 3).A unique feature of this work lies in the consideration of parametric model structures, such as ARMAX, ARARX, BJ and PEM. These models are advocated if there is direct feedback between input and output [4], which is the case when sensors measuring the load and response of a structure are localized in close proximity. Results provided by data-driven parametric model structures, are sufficient to constitute foundations for implementing them as inverse models in the form of fixed-point filters on a DSP platform. The model implemented in such a form is capable of filtering the responses of a mechanical system into a reconstructed input, which is further converted into the frequency domain by a standard DFFT algorithm.

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## IDENTYFIKACJA NIERÓWNOŚCI TORU METODĄ ODWRACANYCH MODELI PARAMETRYCZNYCH

**Streszczenie**: Artykuł przedstawia zastosowanie metody identyfikacji sił obciążających konstrukcje opartej o odwracanie regresyjnych modeli parametrycznych do rekonstrukcji nierówności torów. Nierówności te są identyfikowane na podstawie przebiegów przyspieszeń drgań mierzonych na maźnicach pojazdu podczas jego jazdy. W pracy przedstawiono zarówno weryfikację numeryczną jak i eksperymentalną prezentowanej metody.
**Słowa kluczowe**: identyfikacja nierówności torów, zagadnienie odwrotne, modele parametryczne