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### **DYNAMIC EFFECTS OF CORRUGATIONS**

*We study the rolling motion of a railway wheel on corrugated track. For uniform guiding motion, the trajectory of the contact point, vertical accelerations are evaluated first in the rigid case, then in the case of elastic and visco-elastic contact. Resulting energy dissipation due to friction and percussive effects are calculated and resulting progress of pattern formation is simulated.*

### **EFEKTY DYNAMICZNE SPOWODOWANE PRZEZ KORUGACJE**

*Badane są zjawiska dynamiczne spowodowane ruchem tocznym koła kolejowego po skorugowanym torze. Przy jednostajnym ruchu postępowym osi wyznaczane są trajektorie punktu geometrycznego kontaktu oraz przyspieszenia pionowe, w pierwszej kolejności przypadek brył sztywnych, potem z uwzględnieniem lepko-sprężystego kontaktu tocznego. Dyssypację energii spowodowaną tarciem i efekty uderzeń są obliczane w celu symulacji dalszego procesu formacji korugacji.*

## **1. INTRODUCTION**

Railway wheels should be round and the surface of rails even – without any pattern other than the desired profile. In normal operation, these ideal assumptions are often not fulfilled, small deviations of the nominal shape occur. If such small deviations tend to amplify themselves, or to copy themselves first to the contact partner and propagate in the effect along the track or damage other wheels, considerable damage and inconvenience may arise. Passenger comfort will be reduced and noise radiation increased, more frequent grinding will be necessary [1–3].

Despite intensive studies of longterm effects in rail-wheel contact, the mechanisms of polygonalization and corrugation are still not fully understood [5].

In this article, we give a review of some recent approaches to modeling the essential quantities involved, and we present results of simulations.

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## 2. MODEL COMPONENTS

### 2.1 Geometry

We denote by  $\varphi$  the parameter of the wheel circumference, by  $x$  the position coordinate along the track, by  $z$  the vertical position. In reference configuration, without any perturbations, we have

$$\begin{aligned}x(\varphi) &= R \cos(\varphi) \\z(\varphi) &= R \sin(\varphi)\end{aligned}\tag{1}$$

where  $R$  denotes the nominal radius of the wheel.

On a real railway wheel, we observe small perturbations of the Radius  $R$ , so that in practice, we have  $R = R(\varphi)$ . Typically, the function  $R$  is sinusoidal

$$R(\varphi) = R_0 + a_w \sin(\omega_w \varphi / 2\pi)\tag{2}$$

where  $\omega_w$  is the number of humps around the circumference and  $a_w$  is the amplitude of the deviation.

Similarly, we have patterns on the rail surface, which we describe by

$$z(x) = a_r \sin(\omega_r x / 2\pi)\tag{3}$$

with the number of humps per meter  $\omega_r$ , and the amplitude  $a_r$ .



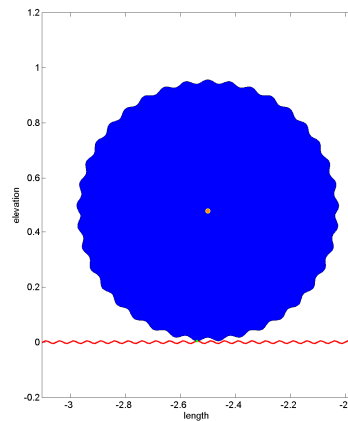
*Fig.1. Photograph of corrugations on a rail.*

Figure 1 presents a typical rail pattern. Of course, several sinusoidal patterns may be superimposed into a trigonometric polynomial or a Fourier series, however, the most interesting case is that of a single dominant pattern. For academic purposes, we study a pair of profiles as in Figure 2, where for better visibility the amplitudes of the pattern have been drastically exaggerated. Actual parameters, as observed by railway operators and used for our calculations, are collected in Table 1.

*Tab. 1. Parameters of geometry perturbations*

parameter	value	unit
$a_r$	1e-4	m
$\omega_r$	10	1/m
$a_w$	1e-4	m
$\omega_w$	37	1
$R_0$	0.47	m

We refer to [4] and [6] for more details and figures.



*Fig. 2 Exaggerated geometrical deviations*

Now, for the special case of an ideal wheel on a sinusoidally perturbed track, the center of the wheel has to satisfy an infinite set of constraints. In fact, unless deformations are allowed, around each point on the track line, a disk of radius  $R_0$  is non-feasible for the wheel's hub position. Assuming continuous contact – without lift-offs – the constraint is constructed in Figure 3. Here we have an invariance in the system, the angle of revolution

of the wheel being a cyclic variable. In the general case of non-ideal shapes of both partners, it is not possible to find a constraint in the 2-d projection of the state space we presented here. Nonetheless, for each  $x$ -position of the hub, and for each angle of revolution, we can determine a minimal elevation  $y$  of the hub, below which penetration of wheel and rail would occur. In the case of ideal bodies, we can additionally determine a unique point of geometrical contact and a common normal direction at that point. In [3], we gave a condition for uniqueness of the geometrical contact point in terms of an inequality between amplitudes and wave numbers  $a_r$ ,  $\omega_r$ ,  $a_w$  and  $\omega_w$ . Without going into detail we repeat here, that for  $R_0$ ,  $\omega_r$  and  $\omega_w$ , as given in Table 1, the amplitudes  $a_r$  and  $a_w$  are restricted to a very small triangle. Otherwise convexity of the wheel is lost, or at least the wheel is no longer *convex enough* to fit into the troughs in the rail surface.

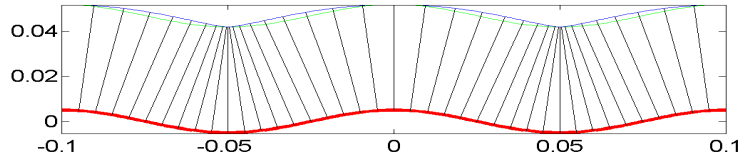


Fig. 3 Track (length in [m]) and constraint (elevation in [m])

The closer we come to the violation of the single-contact-point restriction, the more the constraint becomes curved. Eventually, smoothness is lost.

## 2.1 Forces

Obviously, railway wheels, under the load of their wheelset, bogie, car body and payload, are pressed against the rails, so that a loss of contact seems very unlikely.

On the other hand, there are large masses involved, and their inertia makes it impossible to follow high curvatures of a constraint without extreme reaction forces. This can be seen from the equations of motion (4)

$$\begin{aligned} M\ddot{q}(t) &= f(t, q(t), \dot{q}(t)) \\ 0 &= g(t, q(t)) \end{aligned} \quad (4)$$

where  $q$  denotes the position coordinates,  $t$  denotes time, and superposed dots stand for time derivatives. The functions  $f$  and  $g$  represent the force terms and the constraints, respectively.  $M$  is the mass matrix. Notice that in the general unilateral case, in the second equation of (4), the equality sign has to be replaced by a *smaller than* relation. In [7] and [8], it has been pointed out that even for geometries that allow continuous contact, no realistic normal load suffices to maintain such contact. Hence, jumping of the wheel and plastic impacts have been studied. Figure 4 shows vertical accelerations at a moderate

speed and medium amplitude of corrugations on the rail only. The peaks are due to the nearly non-smooth geometry of the constraint  $g$ , cf. (4.2) and Figure 3.

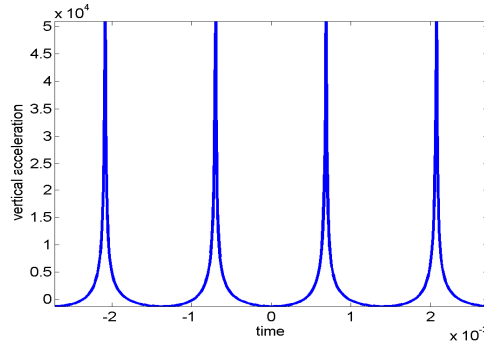


Fig. 4 Vertical accelerations [m/s<sup>2</sup>] vs time [s]

If we do not allow for deformations, neither elastic nor plastic, vertical accelerations as in Figure 4 result from a steady forward motion of the center of the wheel.

**4. NUMERICAL RESULTS**

In the previous Section, we have collected geometrical and dynamical model components of rail-wheel contact in the presence of corrugations. For some academic assumptions, it is possible to calculate contact forces and the speed of the moving load without solving the differential equations (4.1) with algebraic constraints (4.2).

Given the extreme magnitude of the resulting forces, it is obvious that compliance has to be considered. Further, instead of ideal rolling, rather frictional contact has to be assumed for the large tangential forces that would result from a zero difference in tangential velocity in the case of corrugated surfaces.

Consequently, numerical integration has to be applied.

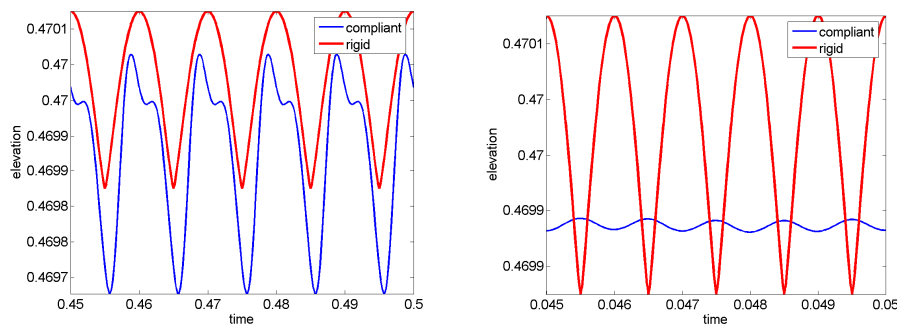


Fig. 5 Vertical hub position [m] vs time [s]

Two typical results for the vertical position coordinate of the wheel center are presented in Figure 5. In the left part, a very slow motion at 5m/s is shown. The normal load is high enough to press both bodies permanently together. The actual elevation is always below the level of the rigid constraint. However, for larger speeds, contact is regularly lost and

reestablished. Surprisingly, the variation of the elevation is diminished at the speed level of 50m/s, Figure 5, right part, despite the percussions due to regular impacts.

### 3. CONCLUSIONS

The development of patterns on contact surfaces is a very complex process. Due to the high number and difficulty of the factors involved, it is still not fully understood. On the other hand, corrugated surfaces themselves are the source – or at least a very important condition – of other effects. Following earlier work of the first author and his group, we studied here the influence of surface imperfections of a certain type on the vertical loading and on the fluctuations in the progressive speed of the moving force. In other work, we analyzed stability regions for supported beams under moving loads. It turns out that corrugations cause – amongst other effects – a repeated slowdown of the longitudinal motion of the load, interleaved with short phases of very high velocity. This means that, repeatedly, the instability curves between regions are crossed. Additionally, the moving force also changes its value. In comparison, changes in the direction of the loading are negligible, as opposed to the case of columns under follower forces. Analytical methods alone cannot assess the impact of mentioned perturbations of the ideal case of uniform motion of a moving load. In a forthcoming paper, numerical calculations of the wave propagation under a variable force at a non-uniform speed are going to be presented. Further, it remains a challenge to take into account the deformability of the contact regions of both bodies – wheel and rail, friction and abrasion, the accumulation of plastic deformations and residual stresses, and hence to study the feedback loop of corrugations back to themselves. In particular, the preference of certain wave lengths, different on rails and on wheels, awaits an explanation in terms of modal instability.

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