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Asphalt-aggregate mixtures,
Rheological models,
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ONE-DIMENSIONAL RHEOLOGICAL MODELS OF ASPHALT-AGGREGATE MIXTURES

Asphalt-aggregate mixtures constitute basic component for the road pavements construction. The fundamental problem in the procedure of pavement design is to elaborate the appropriate constitutive model suited for the structuaral behavior modeling within wide range of mechanical and environmental loadings. This paper deals with the analysis of one-dimensional rheological models of asphalt aggregate-mixtures. We have begun with classical viscoelastic Burgers scheme along with its alternative variants. Then the proposal of an original rheological model including plasticity was given. The differential constitutive relationships of such a model are presented in the paper. The results of computer simulations are also visualized.

JEDNOWYMIAROWE MODELE REOLOGICZNE MIESZANEK MINERALNO-ASFALTOWYCH

Mieszanki mineralno-asfaltowe stanowią podstawowy materiał służący do budowy warstw konstrukcyjnych nawierzchni drogowych. Właściwe zaprojektowanie konstrukcji nawierzchni wymaga opracowania modelu materiału, który odzwierciedla jej zachowanie w szerokim zakresie obciążeń mechanicznych i środowiskowych. W niniejszej pracy są rozpatrywane jednowymiarowe struktury reologiczne wykorzystywane do modelowania konstytutywnych właściwości materiałów warstw bitumicznych. Na początku przypomniano klasyczny, lepkosprężysty model Burgersa wraz z jego alternatywnymi wariantami, a następnie zaprezentowano oryginalną strukturę reologiczną pozwalającą na modelowanie trwałych deformacji o charakterze lepkim i plastycznym. Na jej podstawie sformułowano odpowiednie relacje różniczkowe. Pracę zilustrowano wynikami obliczeń komputerowych.

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1. INTRODUCTION

Nowadays, road constructions are subjected to extremely high traffic loads. Such factors as traffic densities, axle loads and tire pressures are increasing in most countries during the last decades. Thus, the optimization of pavement materials is very important in order to avoid damages and subsequently minimize costs for the road construction and maintenance. On the other hand, pavement failure is not only caused by traffic loads, but is influenced by many other factors like climatic influences and inadequate planning or construction, as well. These factors are able to increase significantly the effects of traffic on the pavement.

In Poland one of the largest infrastructure components are asphalt concrete pavements. Designing of such structures needs realistic constitutive models to be taken into consideration. Developing a realistic mathematical model of asphalt-aggregate mixture is a complicated problem. The complexity is attributed to the time-dependency of the binder, the complex nature of temperature effects, plastic flow of the binder, friction among aggregate particles and coupling the above mentioned effects.

One of the main objectives of pavement research seems to be the prediction of rutting. With the increase of traffic loads and tire pressures, most of the permanent deformation occurs in the upper layers of the road structure rather than in the subgrade. In general, rutting of the asphalt pavement is caused by the combined result of deformation in the non-asphaltic base layers and permanent deformation within the asphalt layers [9].

The elastic behaviour of asphalt-aggregate mixtures combines with its viscous, plastic and fracturing response. Thus, determining the parameters of this material is extremely complicated. Even in such a case when plasticity and fracture are not considered, material viscosity causes temperature and strain rate dependence of the stiffness. For that reason, temperature and strain rate are sometimes integrated in the elastic parameters of the models.

The paper presents one-dimensional constitutive models of asphalt-aggregate mixtures describing their deformation behaviour at a wide range of loading conditions. A special attention will be put on modelling of permanent deformations caused by creep and plasticity. The resulting non-linear models will be mathematically described by the systems of explicit differential equations. An original viscoelastoplastic model based on generalization of the classical Burgers scheme will be presented in details.

2. LINEAR VISCOELASTIC MODELS

Constitutive relationships of linear viscoelastic models can be described by differential or integral equations. Based on the creep or relaxation function it is possible to evaluate the stress or the strain in such a model submitted to force or kinematic excitations, using the following convolution form

$$\begin{aligned} \sigma(t) &= \psi(t) * \dot{\varepsilon}(t) = \int_0^t \psi(t-\tau)\,\mathrm{d}\varepsilon(\tau) = \int_0^t \psi(t-\tau)\dot{\varepsilon}(\tau)\,\mathrm{d}\tau, \\ \varepsilon(t) &= \varphi(t) * \dot{\sigma}(t) = \int_0^t \varphi(t-\tau)\,\mathrm{d}\sigma(\tau) = \int_0^t \varphi(t-\tau)\dot{\sigma}(\tau)\,\mathrm{d}\tau, \end{aligned} \tag{1}$$

where "*" means convolution operation and the functions ψ and φ denote relaxation and creep respectively.

Typical creep and recovery relation for asphaltic materials is shown in Fig. 1. There are three characteristic sections visualized in this graph. The section "a" is equal to immediate, time-independent strains and the section “b” equals the recoverable part of the strain. The “c” section is permanent strain.

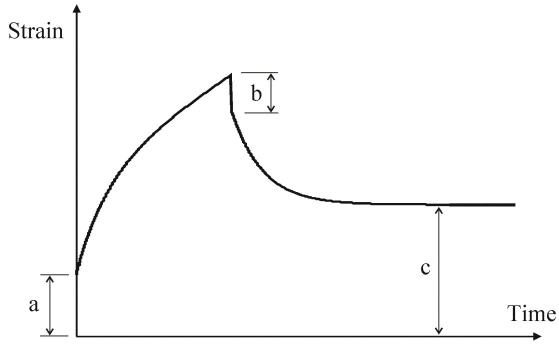

Fig.1. Typical strain-time relation in creep and recovery test

In the case of linear viscoelastic models, time-independent behaviour is always associated with elastic properties of the material while the permanent deformations are associated with viscosity phenomenon. The lengths of sections “a” and “b” are the same assuming viscoelastic behaviour of the material.

Permanent deformations in asphalt layers are first of all the consequence of the viscous bitumen behaviour. A further approach to describe deformations in asphalt layers are rheological models. The main elements of rheological models describing material behaviour are springs and dashpots. Springs represent purely elastic behaviour and dashpots purely viscous behaviour of materials.

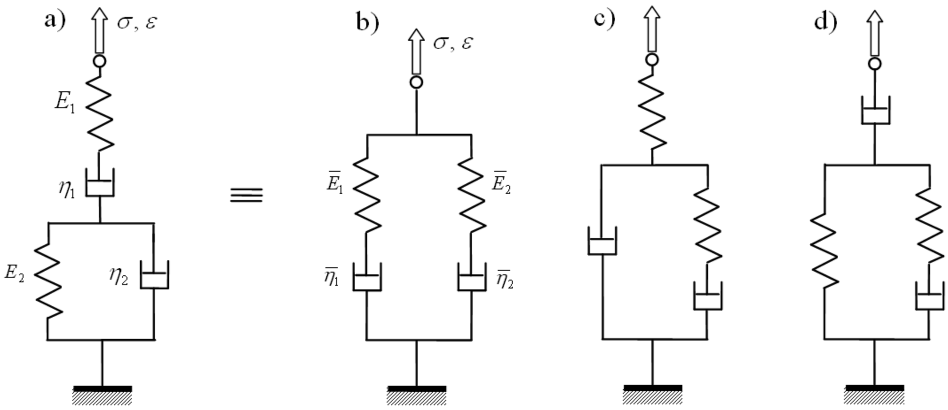

Fig.2. Four equivalent rheological models of viscoelastic material

For the simulation of viscoelastic behaviour springs and dashpots can be combined in two ways, forming the two basic rheological models: the Maxell model contains one spring and one dashpot in series connection, in the Kelvin-Voigt model the two elements are combined in parallel connection. More complex rheological models suited for modeling the effects shown in Fig. 1 (only if $a = b$) are presented in Fig. 2. All the models are composed of 4 linear elements. Mathematically these models are equivalent being described by the same type of linear differential equation [2].

The model shown in Fig. 2a is called Burgers model containing a Maxwell and a Kelvin-Voigt unit in serial connection. Within the Burgers model retardation and relaxation are taken into account. Its relationship is of the following form [1, 7]

$$\sigma + \left(\frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} \right) \dot{\sigma} + \frac{\eta_1 \eta_2}{E_1 E_2} \ddot{\sigma} = \eta_1 \dot{\varepsilon} + \frac{\eta_1 \eta_2}{E_2} \ddot{\varepsilon} \tag{2}$$

The relations between the parameters of the models shown in Fig. 2a (Burgers model) and Fig. 2b (Monismith model) can be expressed in such a form

$$\begin{aligned} E_1 &= \overline{E}_1 + \overline{E}_2, \\ \eta_1 &= \overline{\eta}_1 + \overline{\eta}_2, \\ E_2 &= \frac{\overline{E}_1 \overline{E}_2 \left(\overline{E}_1 + \overline{E}_2 \right) \left(\overline{\eta}_1 + \overline{\eta}_2 \right)^2}{\left(\overline{\eta}_1 \overline{E}_2 - \overline{\eta}_2 \overline{E}_1 \right)^2}, \\ \eta_2 &= \frac{\overline{\eta}_1 \overline{\eta}_2 \left(\overline{\eta}_1 + \overline{\eta}_2 \right) \left(\overline{E}_1 + \overline{E}_2 \right)^2}{\left(\overline{\eta}_1 \overline{E}_2 - \overline{\eta}_2 \overline{E}_1 \right)^2}. \end{aligned} \tag{3}$$

For the linear viscoelastic materials being subjected to steady-state oscillatory forcing conditions it is possible to define the complex modulus in the form

$$E^*(i\omega) = E'(\omega) + i E''(\omega) \tag{4}$$

where E' is the storage modulus, which accounts for the recoverable energy and E'' is the loss modulus, representing the energy dissipation effects. For example, the complex modulus associated with the model shown in Fig. 2b, may be expressed as follows

$$\overline{E}^*(i\omega) = \frac{\overline{E}_1}{1 + \left(i\omega \dfrac{\overline{\eta}_1}{\overline{E}_1} \right)^{-1}} + \frac{\overline{E}_2}{1 + \left(i\omega \dfrac{\overline{\eta}_2}{\overline{E}_2} \right)^{-1}} \tag{5}$$

It should be strongly emphasized that Eq. (5) corresponds to the Monismith model shown in Fig. 2b but do not with the Burgers model visualized in Fig. 2a, as it is sometimes incorrectly presented in the literature (see [3]).

3. VISCOELASTOPLASTIC MODEL

Viscoelastic rheological schemes shown in previous section can only model time-dependent permanent deformations associated with viscosity. Applying large stresses results in additional time-independent plastic deformations. As a consequence the lengths of the sections a and b visualized in Fig. 1 are not the same and obey the relation $a > b$.

We propose the generalization of Burgers model (Fig. 1a) including plasticity. The original rheological model is shown in Fig. 3. The additional elastoplastic network composed of the spring and slider in parallel is used. The limit stress in the slider modeling plasticity is denoted by σ_0 .

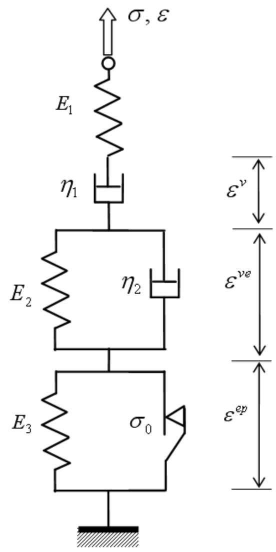

Fig.3. Viscoelastoplastic rheological model of asphalt-aggregate mixture

The total deformation of the body is composed of four parts. The first one is time-independent elastic deformation of the spring E_1 . The second part is viscous permanent deformation of the dashpot η_1 . We introduced an internal variable ε^v in order to describe viscous strains. Another strain variable describing viscoelastic part of the strain is ε^{ve} . Finally the elastoplastic permanent deformations are modeled using the variable ε^{ep} .

The system of constitutive relationships of the proposed model can be formulated in the following form

$$\begin{gathered} \sigma = E_1\left(\varepsilon - \varepsilon^v - \varepsilon^{ve} - \varepsilon^{ep}\right), \\ \sigma^p = \sigma - E_3\varepsilon^{ep}, \\ \dot{\varepsilon}^v = f^v(\sigma), \\ \dot{\varepsilon}^{ve} = f^{ve}\left(\sigma, \varepsilon^{ve}\right), \\ \dot{\varepsilon}^{ep} = f^{ep}\left(\sigma, \sigma^p, \varepsilon^{ve}, \dot{\varepsilon}\right), \end{gathered} \tag{4a}$$

where σ^p denotes plastic stress in the slider. The functions f^v and f^{ve} describing viscous and viscoelastic rates of deformation respectively, is easy to formulate

$$f^{v}(\sigma)=\frac{1}{\eta_1}\sigma,$$
$$f^{ve}(\sigma,\varepsilon^{ve})=\frac{1}{\eta_2}(\sigma-E_2\varepsilon^{ve}). \tag{4b}$$

The crucial problem is to write in an explicit form the elastoplastic strain rate function. In the paper [12] the procedure of so called differential successions was described which may be used for this purpose. The whole algorithm is quite complicated, thus we present only the final result

$$f^{ep}(\sigma,\sigma^p,\varepsilon^{ve},\dot{\varepsilon})=\begin{cases}0 & \text{if} \quad |\sigma^p|<\sigma_0, \\ \dfrac{E_1}{\sigma^p(E_1+E_3)}\left[\sigma^p\left(\dot{\varepsilon}+\dfrac{E_2}{\eta_2}\varepsilon^{ve}-\dfrac{\sigma}{\eta_{eq}}\right)\right]^+ & \text{if} \quad |\sigma^p|=\sigma_0,\end{cases} \tag{4c}$$

where

$$\eta_{eq}:=\frac{\eta_1\eta_2}{\eta_1+\eta_2},\ [z]^+:=\begin{cases}z & \text{if} \quad z>0, \\ 0 & \text{if} \quad z\le 0.\end{cases} \tag{4d}$$

Integrating the system of differential equations (4) we can evaluate the stress history based on the given strain excitations.

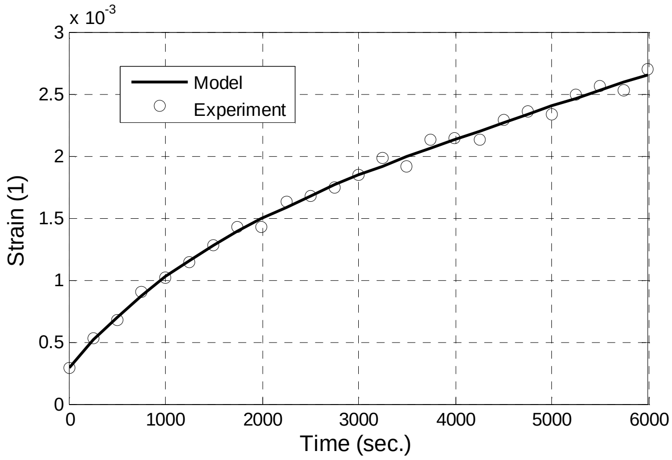

Fig.4. Curve-fitting result for viscoelastic parameters' evaluation

The viscoelastoplastic model being presented in this section depends on six material parameters. The values of viscoelastic components E_1, E_2, η_1 and η_2 can be obtained based on curve fitting procedure of the creep test, because the solution of the Burgers equation (2) with static step excitation has an analytical form [7]. The results of such a procedure using algorithms implemented in MATLAB software are shown in Fig. 4 and in

Table 1. The remaining parameters E_3 and σ_0 may be easily established observing the creep-recovery test results (see Fig. 1). If the applied stress σ is bigger than the plastic limit σ_0, then the following relations hold

$$a = \frac{\sigma - \sigma_0}{E_3} + b\,, \quad b = \frac{\sigma}{E_1}\,. \tag{5}$$

Using Eqs. (5) along with Fig. 1 we can calculate the values of E_3 and σ_0 for certain material. The set of parameters obtained using the above mentioned method is presented in Table 1. The experimental results were assumed based on [4].

Tab. 1. Parameters of the viscoelastoplastic model

E_1 [MPa]	E_2 [MPa]	E_3 [MPa]	η_1 [GPa · s]	η_2 [GPa · s]	σ_0 [MPa]
5130	1650	1540	6160	2150	1,0

Fig. 5 presents numerical results of the creep and recovery simulations. Two models were considered taking the parameter shown in Table 1 – viscoelastic and viscoelastoplastic. The amplitude of step excitation for creep behaviour modeling was equal to 2,0 MPa. The results show how the plastic limit stress influences on the value of intermediate deformations.

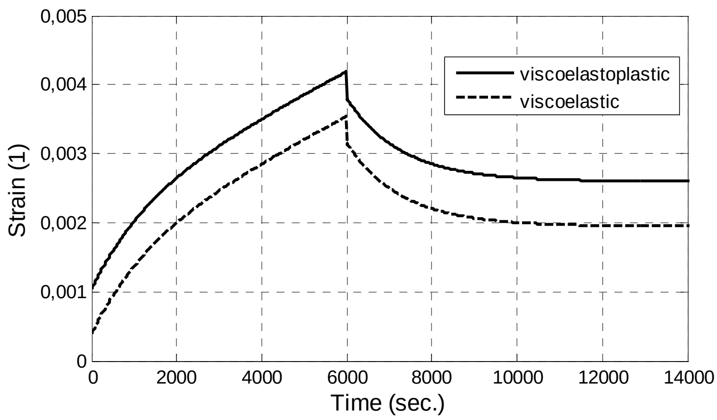

Fig.5. Strain-time relation in creep and recovery numerical test for viscoelastic and viscoelastoplastic models

4. FURTHER INVESTIGATIONS

Three-dimensional generalization of the viscoelastoplastic model presented herein can be obtained based on the procedure explained in [10] and [11]. The choice of an appropriate yield criterion [6, 8] should be justified via experimental tests which are not standard in case of asphalt-aggregate mixtures [5]. Such a model may be used for constitutive description of multilayer road structure subjected to moving load.

5. REFERENCES

[1] Betten J.: *Creep Mechanics*. 2nd Ed., Springer, Berlin 2005.
[2] Bland D. R.: *The Theory of Linear Viscoelasticity.* Pergamon Press, Oxford 1960.
[3] *COST 333: Development of New Bituminous Pavement Design Method - Final Report of the Action.* European Commission Directorate General Transport, Luxembourg, 1999.
[4] Judycki J.: *Modele reologiczne betonu asfaltowego*. Zeszyty Naukowe Politechniki Gdańskiej, nr 368, s. 123-145, Gdańsk 1984.
[5] Kim Y.R.: *Modeling of Asphalt Concrete*. ASCE Press, McGraw-Hill, New York 2009.
[6] Lubarda V.A.: *Elastoplasticity Theory*. CRC, Boca Raton 2002.
[7] Nowacki W. *Teoria pełzania*. Arkady, Warszawa 1963.
[8] Ottosen N.S., Ristinmaa M.: *The Mechanics of Constitutive Modeling*. Elsevier, 2005.
[9] Piłat J., Radziszewski P.: *Nawierzchnie asfaltowe*. Wyd. Komunikacji i Łączności, Warszawa 2003.
[10] Zbiciak A.: *Application of elasto-visco-plastic constitutive model for asphalt pavement creep simulation*. Archives of Civil Engineering, 54, 3, pp. 635-647, 2008.
[11] Zbiciak A.: *Constitutive modelling and numerical simulation of dynamic behaviour of asphalt-concrete pavement*. Engineering Transactions, 56, 4, pp. 311-324, 2008.
[12] Zbiciak A.: *Numerical analysis of dynamic behaviour of elastoplastic beams.* Archives of Civil Engineering, 55, 3, pp. 403-420, 2009.

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